

Nonlinear Dynamo

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Abstract

In this manuscript using the asymptotic method of multiscale non-linear theory we construct a nonlinear theory of the appearance of large-scale structures in the stratified conductive medium with the presence of small-scale oscillations of the velocity field and magnetic fields. These small-scale stationary oscillations are maintained by small external sources at low Reynolds numbers. We obtain a nonlinear system of equations describing the evolution of large-scale structures of the velocity field and magnetic fields. The linear stage of evolution leads to the known instability. In this article we consider the stationary large-scale structures of a magnetic field arising at stabilization of linear instability.

1 Introduction

It is known, that the term dynamo originated in theory of the generation of large-scale magnetic fields in magnetic hydrodynamics, mainly in its applications to astrophysics and geophysics. The main mechanism of the generation

of large-scale magnetic fields (i.e., the dynamo mechanism) is a violation of the parity of fluid velocity field. In particular, a violation of parity leads to a non-zero velocity field helicity $\alpha = \langle \vec{V} \text{rot} \vec{V} \rangle$ (α -effect). Here α is pseudo scalar. More generally, the violation of parity gives AKA effect (anisotropic kinetic effect). In turn, these effects generate large-scale magnetic field instability and generate large-scale vortex instability of fluid. In recent years, this effect got the name of vortex dynamo. Currently the theory of magnetic dynamo is well developed in the kinematic formulation, when we assume that the velocity field is given. This theory is presented in numerous articles and books (e.g. [1], [2], [3], [4], [5], [6]). The theory of the vortex dynamo is younger, but it is also the subject of many papers (e.g. [7]-[10]). The main application of the vortex dynamo theory is the generation of localized vortex structures in the atmosphere, for example, such as tropical cyclones [8], [10]. Actually the large-scale vortex structures can be generated by the same dynamo mechanism in the conductive fluid, like the magnetic field. The question then, is how these vortex structures coexist with the magnetic structures and what is the nonlinear dynamics of magnetic and vortex structures. These issues are the subject of this article.

In this paper, we consider that a small-scale external force with a nonzero helicity operates in the conductive fluid, which engenders the helical velocity field fluctuations, i.e. models helical turbulence. Note that we understand under the large scale structure the structures which the characteristic scale is much larger than the scale of external force or of turbulence which generates them. As a mathematical technique which allows to study the nonlinear dynamo we use the method of multiscale developments proposed in [11]. In this method, the asymptotic development is carried out according to the small Reynolds number $R \ll 1$. This makes possible to obtain the closed equations for large-scale fields and study the linear and nonlinear instabilities. As a result, we observe a large number of different nonlinear vortices and magnetic structures: nonlinear waves, solitons, kinks and chaotic structures.

Strictly speaking, this theory is applicable at small Reynolds numbers, i.e. for motion of conducting fluid with a sufficiently high viscosity, for example a liquid core of the Earth. Actually in astrophysics, the Reynolds number is usually large and the motion of conducting fluid is turbulent. If we use the turbulent viscosity to estimate the Reynolds number, the effective Reynolds number can be reduced to a value of the order of unity. It enables to use the theory presented in this article for qualitative estimations. Some examples of such assessments to the photosphere of the Sun will be given further.

The paper is organized as follows. In Sec.2 are set out the basic equations and the formulation of the problem. In Sec.3 are derived the basic equations of the nonlinear dynamo. In Sec.4 is described the large-scale linear instability. In Sec.5 are described the stationary nonlinear magnetic structures. In the appendices are described various technical issues, such as multi-scale asymptotic developments and the closure of the Reynolds stresses.

2 Basic equations and formulation of the problem

Let us consider equations of motion of the incompressible electroconducting medium with a constant temperature gradient in Boussinesq approximation [12]:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \nabla) \vec{V} = -\frac{\nabla P}{\rho_{00}} + \nu \Delta \vec{V} + \frac{1}{4\pi\rho_{00}} [\text{rot} \vec{B} \times \vec{B}] + g\beta T \vec{e} + \vec{F}_0, \quad (1)$$

$$\frac{\partial T}{\partial t} + (\vec{V} \nabla) T = \chi \Delta T - V_z A, \quad (2)$$

where $\vec{e} = (0, 0, 1)$ is the unit vector in the direction of the axis z , β is the thermal development coefficient, $A = \frac{dT_{00}}{dz}$ is the constant temperature gradient, $A = \text{const}$, $A > 0$. To the equations (1)-(2) should be added the equations for magnetic induction \vec{B} and solenoidality conditions for fields \vec{V} and \vec{B} :

$$\frac{\partial \vec{B}}{\partial t} = \text{rot} [\vec{V} \times \vec{B}] + \nu_m \Delta \vec{B} \quad (3)$$

$$\text{div} \vec{B} = 0, \quad \text{div} \vec{V} = 0 \quad (4)$$

Here, $\nu_m = \frac{c^2}{4\pi\sigma}$ is the coefficient of magnetic viscosity, σ is the electrical conductivity of the medium. The system of equations (1)-(4) describes the evolution of disturbances on the background of the equilibrium state defined by the equilibrium condition:

$$\nabla P_{00} = \rho_{00} g \beta T_{00} \quad (5)$$

$\rho_{00} = \text{const}$ is the density of medium, χ is the thermal conductivity of the medium. We chose these unusual designations for an equilibrium state to

avoid further any confusion with asymptotic development. Equation (1) contains the external force \vec{F}_0 which has the following property:

$$\vec{F}_0 = f_0 \vec{F}_0 \left(\frac{x}{\lambda_0}; \frac{t}{t_0} \right), \quad \text{div} \vec{F}_0 = 0, \quad \vec{F}_0 \cdot \text{rot} \vec{F}_0 \neq 0 \quad (6)$$

where λ_0 - the characteristic scale, t_0 - the characteristic time, f_0 - the characteristic amplitude of the force. The main role of this force is to create the helical velocity field \vec{v}_0 with a low Reynolds number $R = \frac{v_0 t_0}{\lambda_0} \ll 1$ in the medium of small-scale fluctuations. In other words, to maintain the small-scale helical turbulence. It is easy to notice that the characteristic velocity v_0 , generated by an external force, has the same characteristic scales:

$$v_0 = v_0 \left(\frac{x}{\lambda_0}, \frac{t}{t_0} \right) \quad (7)$$

Now we write the system of equations (1)-(4) in dimensionless variables:

$$\vec{x} \rightarrow \frac{\vec{x}}{\lambda_0}, \quad t \rightarrow \frac{t}{t_0}, \quad \vec{v}_0 \rightarrow \frac{\vec{v}_0}{v_0}, \quad \vec{F}_0 \rightarrow \frac{\vec{F}_0}{f_0}, \quad P \rightarrow \frac{P}{\rho_{00} P_0}, \quad (8)$$

$$\vec{B} \rightarrow \frac{\vec{B}}{B_0}, \quad t_0 = \frac{\lambda_0^2}{\nu}, \quad P_0 = \frac{v_0 \nu}{\lambda_0^2}, \quad T \rightarrow \frac{T}{\lambda_0 A}$$

In these variables, equations (1)-(4) take the form:

$$\frac{\partial \vec{V}}{\partial t} + R \left(\vec{V} \nabla \right) \vec{V} = -\nabla P + \Delta \vec{V} + \widetilde{Ra} T \vec{e} + \widetilde{Q} R \left[\text{rot} \vec{B} \times \vec{B} \right] + \vec{F}_0 \quad (9)$$

$$\frac{\partial T}{\partial t} - Pr^{-1} \Delta T = -R \left(\vec{V} \nabla \right) T - V_z \quad (10)$$

$$\frac{\partial \vec{B}}{\partial t} - Pm^{-1} \Delta \vec{B} = R \text{rot} \left[\vec{V} \times \vec{B} \right] \quad (11)$$

where $\widetilde{Ra} = \frac{Ra}{Pr}$, $Ra = \frac{g \beta A \lambda_0^4}{\nu \chi}$ is the Rayleigh number on the scale of λ_0 ; $Pr = \frac{\nu}{\chi}$ is the Prandtl number; $\widetilde{Q} = \frac{Q}{Pm}$, $Q = \frac{\sigma B_0^2 \lambda_0^2}{c^2 \rho_{00} \nu}$ is the Chandrasekhar number of scale λ_0 ; $Pm = \frac{\nu}{\nu_m}$ is the magnetic Prandtl number; B_0 - the characteristic small-scale magnetic field, which we consider as initial small magnetic field [3]). Let us consider as the small parameter of the asymptotic development the Reynolds number R of the small-scale turbulence. Smallness

of the remaining parameters is not supposed and the parameters of Ra and Q , do not influence the scheme of asymptotic development.

We turn now to the next formulation of the problem. Let the external force F_0 be helical, small-scale and high-frequency. This force engenders the small-scale fluctuations of velocity and temperature compared to the equilibrium state. Small-scale fluctuations of a magnetic field are generated by non turbulent mechanisms, for example, by means of thermo-effects [13], [6], plasma instabilities [14], [15], etc. When averaging the rapidly oscillating small-scale fluctuations give zero. However, due to the nonlinear interaction between them, there may be terms which do not vanish in the averaging. These terms are called secular, and they are condition of solvability of multi-scale asymptotic development. So the main problem is to find and study the solvability equations, i.e., the equations for large-scale perturbations.

3 Equations of non-linear dynamo in "quasi two-dimensional" model

Let us consider in more detail the application of the method of multi-scale asymptotic development to the problem of nonlinear evolution of the large-scale vortex and magnetic perturbations in the convective and electrically conductive medium. In order to construct the multi-scale asymptotic developments we will use the methods of [16], [17]. Let us denote the small-scale variables $x_0 = (\vec{x}_0, t_0)$, and the large-scale $X = (\vec{X}, T)$. The derivative of $\frac{\partial}{\partial x_0^i}$ is denoted by ∂_i , and the derivative $\frac{\partial}{\partial t_0}$ as ∂_t . Further, the large-scale spatial and temporal derivatives will be denoted as:

$$\frac{\partial}{\partial X_i} \equiv \nabla_i,$$

$$\frac{\partial}{\partial T} \equiv \partial_T.$$

In accordance with the method of multiple scales represent the spatial and temporal derivatives in equations (9)-(11) in the form of derivatives of the small-scale and large-scale variables:

$$\frac{\partial}{\partial x_i} \rightarrow \partial_i + R^2 \nabla_i \tag{12}$$

$$\frac{\partial}{\partial t} \rightarrow \partial_t + R^4 \partial_T \quad (13)$$

Now the perturbed fields \vec{V} , T , \vec{B} and P we develop into series over the small parameter R and we obtain:

$$\begin{aligned} \vec{V}(\vec{x}, t) &= \frac{1}{R} W_{-1}(X) + \vec{v}_0(x_0) + R\vec{v}_1 + R^2\vec{v}_2 + R^3\vec{v}_3 + \dots \\ T(\vec{x}, t) &= \frac{1}{R} T_{-1}(X) + T_0(x_0) + RT_1 + R^2T_2 + R^3T_3 + \dots \\ \vec{B}(\vec{x}, t) &= \frac{1}{R} \vec{B}_{-1}(X) + \vec{B}_0(x_0) + R\vec{B}_1 + R^2\vec{B}_2 + R^3\vec{B}_3 + \dots \\ P(\vec{x}, t) &= \frac{1}{R^3} P_{-3} + \frac{1}{R^2} P_{-2} + \frac{1}{R} P_{-1} + P_0 + R(P_1 + \overline{P}_1(X)) + R^2 P_2 + R^3 P_3 + \dots \end{aligned} \quad (14)$$

Here the contribution of P_1 depends on small-scale variables and $\overline{P}_1(X)$ only on large scale. Substituting (12)-(14) in the system of equations (9)-(11) and putting together terms of the same order for the R including R^3 , we obtain a system of equations of multi-scale asymptotic developments. The main problem is to isolate from these equations the secular conditions which determine the dynamics of disturbances on a large scale. The algebraic structure of the asymptotic development of the equations (9)-(11) in different orders of R is given in Appendix I. It is also shown that the main secular equation, i.e. equation for large-scale fields is obtained in the order R^3 :

$$\partial_T W_i - \Delta W_i + \nabla_k \overline{(v_0^k v_0^i)} = -\nabla_i \overline{P} + \tilde{Q} \left(\nabla_k \left(\overline{B_0^i B_0^k} \right) - \frac{\nabla_i}{2} \left(\overline{B_0^k} \right)^2 \right) \quad (15)$$

$$\partial_T H_i - P m^{-1} \Delta H_i = \nabla_j \left(\overline{v_0^i B_0^j} \right) - \nabla_j \left(\overline{v_0^j B_0^i} \right) \quad (16)$$

$$\partial_T \Theta - P r^{-1} \Delta \Theta + \nabla_k \left(\overline{v_0^k T_0} \right) = 0 \quad (17)$$

Secular equations, derived in Appendix I are added to the Equations (15)-(17):

$$\nabla_k (W_k W_i) = -\nabla_i \overline{P}_{-1} + \tilde{Q} (\nabla_k H_i - \nabla_i H_k) H_k$$

$$\nabla_k (W_k \Theta) = 0$$

$$W_j \nabla_j H_i = H_j \nabla_j W_i$$

$$\nabla_i W_i = 0, \quad \nabla_i H_i = 0, \quad W_z = 0$$

Thus, to obtain equations (15)-(16), which describes the evolution of large-scale fields \vec{W} and \vec{H} it is necessary to reach the third order of perturbation theory. This is a quite typical phenomenon when applying the method of multiscale developments. Equations (15)-(16) become closed after calculation of the correlation functions of Reynolds stress:

$$\overline{v_0^k v_0^i} = \overline{v_{01}^k (v_{01}^i)^*} + \overline{(v_{01}^k)^* v_{01}^i} + \overline{v_{03}^k (v_{03}^i)^*} + \overline{(v_{03}^k)^* v_{03}^i} = T_{(1)}^{ki} + T_{(2)}^{ki},$$

Maxwell stresses:

$$\overline{B_0^i B_0^k} = \overline{B_{01}^i (B_{01}^k)^*} + \overline{(B_{01}^i)^* B_{01}^k} + \overline{B_{03}^i (B_{03}^k)^*} + \overline{(B_{03}^i)^* B_{03}^k} = S_{(1)}^{ik} + S_{(2)}^{ik},$$

and correlators entering in the definition of turbulent e.m.f $\mathcal{E}_n = \varepsilon_{nij} \overline{v_0^i B_0^j}$ (see for example [3]):

$$\overline{v_0^i B_0^j} = \overline{v_{01}^i (B_{01}^j)^*} + \overline{(v_{01}^i)^* B_{01}^j} + \overline{v_{03}^i (B_{03}^j)^*} + \overline{(v_{03}^i)^* B_{03}^j} = G_{(1)}^{ij} + G_{(2)}^{ij}.$$

Complex conjugate values here and later will be designated by an asterisk. The "quasi two-dimensional" approach is often used in many astrophysical and geophysical problems to describe the dynamics of large-scale vortex and magnetic fields [18], [16], [17], [6]. In the frame of this approximation, we assume that the large-scale derivative over Z is much more than others derivatives, i.e.,

$$\frac{\partial}{\partial Z} \gg \frac{\partial}{\partial X}, \frac{\partial}{\partial Y},$$

and the geometry of the large-scale fields is as follows:

$$\vec{W} = (W_1(Z), W_2(Z), 0), \quad \vec{H} = (H_1(Z), H_2(Z), 0) \quad (18)$$

This geometry corresponds to the large-scale field (Beltrami field): $\vec{W} \times \text{rot} \vec{W} = 0$ and $\vec{H} \times \text{rot} \vec{H} = 0$. Finally, taking into account the geometry of the problem (18), the equation for large-scale disturbances take the form:

$$\partial_T W_1 - \nabla_Z^2 W_1 + \nabla_Z (\overline{v_0^z v_0^x}) = \tilde{Q} \nabla_Z (\overline{B_0^z B_0^x}) \quad (19)$$

$$\partial_T W_2 - \nabla_Z^2 W_2 + \nabla_Z (\overline{v_0^z v_0^y}) = \tilde{Q} \nabla_Z (\overline{B_0^z B_0^y}) \quad (20)$$

$$\partial_T H_1 - P m^{-1} \nabla_Z^2 H_1 = \nabla_Z (\overline{v_0^x B_0^z}) - \nabla_Z (\overline{v_0^z B_0^x}) \quad (21)$$

$$\partial_T H_2 - Pm^{-1} \Delta H_2 = \nabla_Z \left(\overline{v_0^y B_0^z} \right) - \nabla_Z \left(\overline{v_0^z B_0^x} \right) \quad (22)$$

$$\partial_T \Theta - Pr^{-1} \nabla_Z^2 \Theta + \nabla_Z \left(\overline{v_0^z T_0} \right) = 0 \quad (23)$$

$$\widetilde{Ra} \Theta e_z = \nabla_Z P_{-3}, \quad \nabla_Z \equiv \frac{\partial}{\partial Z} \quad (24)$$

For equations (19)-(22) in closed form, we use solutions of equations for small-scale fields in zero order of R , obtained in Appendix II. Further it is necessary to calculate the correlators included in the system of equations (19) - (22). The technical aspect of this issue is described in detail in Annex III. in Appendix III. As a result of these the calculations of components $T_{(2)}^{31}$, $T_{(1)}^{32}$, $S_{(2)}^{31}$, $S_{(1)}^{32}$, $\delta G_{(2)} = G_{(2)}^{13} - G_{(2)}^{31}$, $\delta G_{(1)} = G_{(1)}^{23} - G_{(1)}^{32}$, we get a closed equation for large scale velocity fields (W_1, W_2) and magnetic field (H_1, H_2) in the following form:

$$\partial_T W_1 - \nabla_Z^2 W_1 - \nabla_Z \left[\alpha^{(2)} \cdot (1 - W_2) \left(1 - \frac{H_2^2 Pm Q}{(1 + Pm^2(1 - W_2)^2)} \right) \right] = 0 \quad (25)$$

$$\partial_T W_2 - \nabla_Z^2 W_2 + \nabla_Z \left[\alpha^{(1)} \cdot (1 - W_1) \left(1 - \frac{H_1^2 Pm Q}{(1 + Pm^2(1 - W_1)^2)} \right) \right] = 0 \quad (26)$$

$$\partial_T H_1 - Pm^{-1} \nabla_Z^2 H_1 - \nabla_Z \left(\alpha_H^{(2)} \cdot H_2 \right) = 0 \quad (27)$$

$$\partial_T H_2 - Pm^{-1} \nabla_Z^2 H_2 + \nabla_Z \left(\alpha_H^{(1)} \cdot H_1 \right) = 0 \quad (28)$$

Equations (25)-(28) describe the nonlinear dynamics of the large-scale fields in electroconductive medium with temperature inhomogeneity. Connection between the components of a large-scale vortex and magnetic field is carried out by means of the coefficients of the nonlinear hydrodynamic (HD) $\alpha^{(1)}$, $\alpha^{(2)}$ and magnetohydrodynamic (MHD) $\alpha_H^{(1)}$, $\alpha_H^{(2)}$ α -effect. Moreover, the coefficients of nonlinear HD and MHD α -effect are functions of large-scale velocity fields \vec{W} and the magnetic field \vec{H} :

$$\begin{aligned} \alpha^{(1)} = & \frac{\widetilde{Ra}(1 + Pm^2 \widetilde{W}_1^2) \left[(1 + Pr)(1 + Pm^2 \widetilde{W}_1^2) + QH_1^2(Pr - Pm) \right]}{2 \left[(1 - Pm \widetilde{W}_1^2 + QH_1^2)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right]} \times \\ & \times \left[\left((1 - Pm \widetilde{W}_1^2 + QH_1^2)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right) (1 + Pr^2 \widetilde{W}_1^2) + \right. \end{aligned}$$

$$+2Ra \left(\left(1 - Pr\widetilde{W}_1^2\right) \left(1 + Pm^2\widetilde{W}_1^2\right) + QH_1^2 \left(1 + Pm\widetilde{W}_1^2\right) \right) + \\ + Ra^2(1 + Pm^2\widetilde{W}_1^2) \Big]^{-1}, \quad (29)$$

$$\alpha^{(2)} = \frac{\widetilde{Ra}(1 + Pm^2\widetilde{W}_2^2) \left[\left(1 + Pr\right)(1 + Pm^2\widetilde{W}_2^2) + QH_2^2(Pr - Pm) \right]}{2 \left[\left(1 - Pm\widetilde{W}_2^2 + QH_2^2\right)^2 + \widetilde{W}_2^2(1 + Pm)^2 \right]} \times \\ \times \left[\left(\left(1 - Pm\widetilde{W}_2^2 + QH_2^2\right)^2 + \widetilde{W}_2^2(1 + Pm)^2 \right) \left(1 + Pr^2\widetilde{W}_2^2\right) + \right. \\ \left. + 2Ra \left(\left(1 - Pr\widetilde{W}_2^2\right) \left(1 + Pm^2\widetilde{W}_2^2\right) + QH_2^2 \left(1 + Pm\widetilde{W}_2^2\right) \right) + \right. \\ \left. + Ra^2(1 + Pm^2\widetilde{W}_2^2) \right]^{-1}, \quad (30)$$

$$\alpha_H^{(1)} = \frac{Pm}{\left(\left(1 - Pm\widetilde{W}_1^2 + QH_1^2\right)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right)} \left\{ 1 - \right. \\ \left. - Ra \left[\left(1 - Pr\widetilde{W}_1^2\right) + \frac{QH_1^2(1 + PrPm\widetilde{W}_1^2)}{(1 + Pm^2\widetilde{W}_1^2)} + Ra \right] \times \right. \\ \times \left[\left(\left(1 - Pm\widetilde{W}_1^2 + QH_1^2\right)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right) \frac{(1 + Pr^2\widetilde{W}_1^2)}{(1 + Pm^2\widetilde{W}_1^2)} + \right. \\ \left. + 2Ra \left[\left(1 - Pr\widetilde{W}_1^2\right) + \frac{QH_1^2(1 + PrPm\widetilde{W}_1^2)}{(1 + Pm^2\widetilde{W}_1^2)} \right] + Ra^2 \right]^{-1} \Big\}, \quad (31)$$

$$\alpha_H^{(2)} = \frac{Pm}{\left(\left(1 - Pm\widetilde{W}_2^2 + QH_2^2\right)^2 + \widetilde{W}_2^2(1 + Pm)^2 \right)} \times \\ \times \left\{ 1 - Ra \left[\left(1 - Pr\widetilde{W}_2^2\right) + \frac{QH_2^2(1 + PrPm\widetilde{W}_2^2)}{(1 + Pm^2\widetilde{W}_2^2)} + Ra \right] \times \right. \\ \times \left[\left(\left(1 - Pm\widetilde{W}_2^2 + QH_2^2\right)^2 + \widetilde{W}_2^2(1 + Pm)^2 \right) \frac{(1 + Pr^2\widetilde{W}_2^2)}{(1 + Pm^2\widetilde{W}_2^2)} + \right.$$

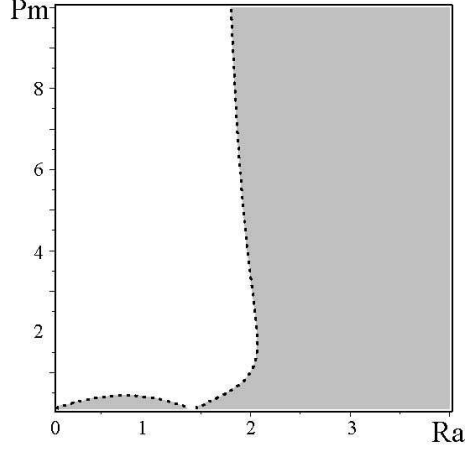


Figure 1: On the the parameter plane (Ra, Pm) the gray area shows the $\gamma > 1$ and the white shows the region $\gamma < 1$.

$$+2Ra \left[\left(1 - Pr\widetilde{W}_2^2 \right) + \frac{QH_2^2 \left(1 + PrPm\widetilde{W}_2^2 \right)}{\left(1 + Pm^2\widetilde{W}_2^2 \right)} + Ra^2 \right]^{-1} \Bigg\} \quad (32)$$

Here we use the notation $\widetilde{W}_1 = 1 - W_1$, $\widetilde{W}_2 = 1 - W_2$ in order to shorten the recording of formulas (29)-(32). Let us note, that HD nonlinear α -effect described by the system of equations (25)-(26), is possible in presence of temperature stratification, i.e., Rayleigh number of $Ra \neq 0$ and the external helical forces $\vec{f}_0 \neq 0$. On the contrary, a MHD nonlinear α -effect occurs, when there is no heating of $Ra = 0$. In the non electroconductive medium with $\sigma = 0$, the equation (25)-(26) coincide with the results of [16], [17]. Like in [17], we consider first of all the stability of the small field disturbances (linear theory), and then examine the possibility of the existence of stationary structures.

4 Large-scale instability

Let us consider the initial stage of development of the perturbations (W_1, W_2) and (H_1, H_2) . Then, for small values of W_1, W_2 and H_1, H_2 the equations (25)-(28) are linearized and can be reduced to the following system of linear

equations:

$$\partial_T W_1 - \nabla_Z^2 W_1 + \alpha \nabla_Z W_2 = 0 \quad (33)$$

$$\partial_T W_2 - \nabla_Z^2 W_2 - \alpha \nabla_Z W_1 = 0 \quad (34)$$

$$\partial_T H_1 - \nabla_Z^2 H_1 - \alpha_H \nabla_Z H_2 = 0 \quad (35)$$

$$\partial_T H_2 - \nabla_Z^2 H_2 + \alpha_H \nabla_Z H_1 = 0 \quad (36)$$

where

$$\alpha = -\frac{Ra(4 - 2Ra)}{(4 + Ra^2)^2} \quad \text{for numbers} \quad Pr = 1 \quad (37)$$

$$\alpha_H = \frac{2Pm}{(1 + Pm^2)(4 + Ra^2)} \quad (38)$$

From the equations (33)-(36) we can see that under small perturbations of fields the self-consistent system of equations (25)-(28) splits into two pairs of equations for large-scale field \vec{W} and magnetic field \vec{H} respectively. The first pair of equations (33)-(34) are similar to the hydrodynamic equations for α -effect [7], [9], which generate large-scale vortex structures. The second pair of equations (35)-(36) describes a well-known theory of the dynamo [3] - [5], [19], the α -effect amplifies the large-scale magnetic field by the small-scale helical turbulence. In the linear theory considered here, the generation coefficients α and α_H do not depend on fields amplitudes but depend only on the characteristics of the medium. For the study of large-scale instability, described by the equations (33)-(36), we choose perturbations of velocity (W_1, W_2) and magnetic induction (H_1, H_2) in the form of flat circular polarized waves:

$$W_1 = A_W \exp(\Gamma t) \sin KZ, \quad W_2 = B_W \exp(\Gamma t) \cos KZ \quad (39)$$

$$H_1 = A_H \exp(\Gamma t) \sin KZ, \quad H_2 = B_H \exp(\Gamma t) \cos KZ \quad (40)$$

Substituting (39) in the system of equations (33)-(34), and (40) in (35)-(36), we obtain two instability increments:

$$\Gamma_1 = \pm \alpha K - K^2 \quad (41)$$

$$\Gamma_2 = \pm \alpha_H K - K^2 Pm^{-1} \quad (42)$$

The growing solution with the first increment, describes the generation of helical vortex structures of Beltrami type. The maximum of increment

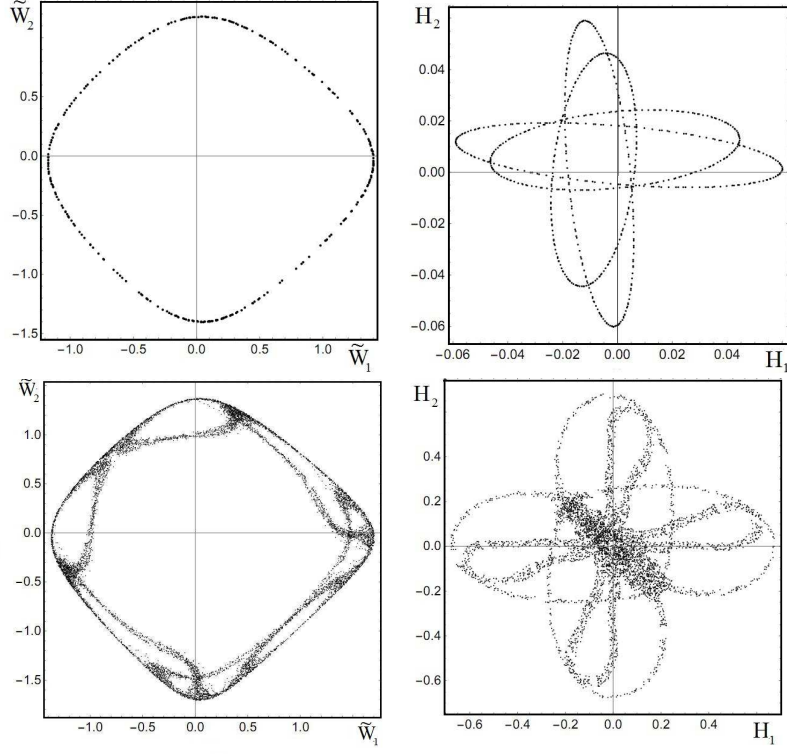


Figure 2: Poincaré section for two trajectories. The upper, for the trajectory with the initial conditions $\tilde{W}_1(0) = 0.8$, $\tilde{W}_2(0) = 0.8$, $H_1(0) = 0.01$, $H_2(0) = 0.01$ and the lower with the initial conditions $\tilde{W}_1(0) = 0.9$, $\tilde{W}_2(0) = 0.9$, $H_1(0) = 0.01$, $H_2(0) = 0.01$. For the left figures, the cutting plane is stretched to single coordinate vectors of the velocity, and for the right ones to coordinate vectors of the magnetic field. One can see that the trajectory corresponding to the upper figures is wound on tori. These trajectories are regular. The lower figures show stochastic layers. The corresponding chaotic trajectories belong to them.

$\Gamma_{1max} = \frac{\alpha^2}{4}$ is reached at $K_{max} = \frac{\alpha}{2}$. In the same way, we find from formula (42) the maximum increment of large-scale magnetic field generation $\Gamma_{2max} = \frac{\alpha_H^2}{4} Pm$ at $K_{max} = \frac{\alpha_H}{2} Pm$.

If the external force has zero helicity $\vec{F}_0 \text{rot} \vec{F}_0 = 0$, the two α -effects disappear: $\alpha = 0$, $\alpha_H = 0$. In addition, when the temperature gradient is vanishing, the hydrodynamic α -effect also disappears, but MHD α -effect remains. So in these conditions, the magnetic field continues to grow. In order to understand for which Rayleigh number Ra are generated large-scale vortices or magnetic disturbances it is convenient to introduce the coefficient of relative growth rate of perturbations $\gamma = \Gamma_{1max}/\Gamma_{2max}$:

$$\gamma = \frac{\Gamma_{1max}}{\Gamma_{2max}} = \frac{Ra^2(4 - 2Ra)^2(1 + Pm^2)^2}{4Pm^3(4 + Ra^2)^2} \quad (43)$$

From this we can see that at small magnetic Prandtl $Pm \ll 1$ and Rayleigh numbers of $Ra > 2$ there is the most effective generation of large-scale vortex disturbances. The small magnetic Prandtl numbers of Pm can be due to the low electrical conductivity ($\sigma \rightarrow 0$) medium and very low kinematic viscosity ($\nu \rightarrow 0$) of medium. Now consider the case of low electroconductivity medium ($\sigma \rightarrow 0$) with $\nu \neq 0$, then the magnetic number of Prandtl and Chandrasekhar number are small: $Pm \rightarrow 0$, $Q \rightarrow 0$. In this medium, the generation of large-scale magnetic field is not effective $H_{1,2} \ll 1$ because the coefficient $\gamma \gg 1$, and under the influence of external helical small-scale force the generation of large-scale vortex structures is possible in the convective medium which are described in detail in works [16], [17]. We turn now to the case of an electrically conductive medium, i.e., when the magnetic number of Prandtl is non zero $Pm \neq 0$. From the numerical analysis of the formula (43) (see fig.1) it follows that the generation of vortex disturbances is most efficient when Rayleigh numbers are $Ra > 2$. White area in the figure1 shows the area of the preferential generation of magnetic disturbances. We can state that the magnetic field generation is most effective for Rayleigh numbers in the interval $Ra \in [0, 2]$. The linear theory is not correct when the large-scale perturbations are strong enough. Therefore it is necessary to take into account the nonlinear effects.

5 Stationary nonlinear magnetic structures

We turn now to a discussion of the nonlinear stage. Taking into account the dependence of the right sides of the system of nonlinear equations of \vec{W} , \vec{H} , one would expect that with the growth of perturbations the nonlinear coefficients of $\alpha^{(1)}$, $\alpha^{(2)}$, $\alpha_H^{(1)}$, $\alpha_H^{(2)}$ decrease and the instability is stabilized. As a result the nonlinear stationary structures are formed. To describe these structures let us examine the nonlinear system of equations (25) - (28) in the stationary case, taking $\partial_T W_1 = \partial_T W_2 = \partial_T H_1 = \partial_T H_2 = 0$. Integrating these equations on Z we obtain:

$$\frac{d\widetilde{W}_1}{dZ} = \alpha^{(2)}\widetilde{W}_2 \left(1 - \frac{QH_2^2 Pm}{1 + Pm^2\widetilde{W}_2^2} \right) + C_1 \quad (44)$$

$$\frac{d\widetilde{W}_2}{dZ} = -\alpha^{(1)}\widetilde{W}_1 \left(1 - \frac{QH_1^2 Pm}{1 + Pm^2\widetilde{W}_1^2} \right) + C_2 \quad (45)$$

$$\frac{1}{Pm} \frac{dH_1}{dZ} = -\alpha_H^{(2)} H_2 + C_3 \quad (46)$$

$$\frac{1}{Pm} \frac{dH_2}{dZ} = \alpha_H^{(1)} H_1 + C_4 \quad (47)$$

Here C_1, C_2, C_3, C_4 are arbitrary constants of integration. Equations (44) - (47) present the nonlinear dynamical system in four-dimensional phase space. It can be proved that this system of equations is conservative. However to find the Hamiltonian of this nonlinear system is technically bulky task. Even if it exists, it can only be obtained in quadratures and the execution of integration removes it beyond the class of elementary functions.

In general case this conservative nonlinear system of four coupled equations has no attractors in phase space. The high dimensionality of the phase space and a large number of parameters in the system make very difficult the full qualitative analysis of this system. One of the common features is the invariance of the system under the transformation $(\widetilde{W}_1, \widetilde{W}_2, H_1, H_2) \rightarrow (-\widetilde{W}_1, -\widetilde{W}_2, -H_1, -H_2)$. With zero constant $C_1 = C_2 = C_3 = C_4 = 0$ in the phase space a fixed point exists at the origin of the phase space. In the phase space of such a system the presence of resonant and nonresonant tori can be expected. In its turn this means the existence of chaotic stationary structures of hydrodynamic and magnetic fields. To prove the existence of these

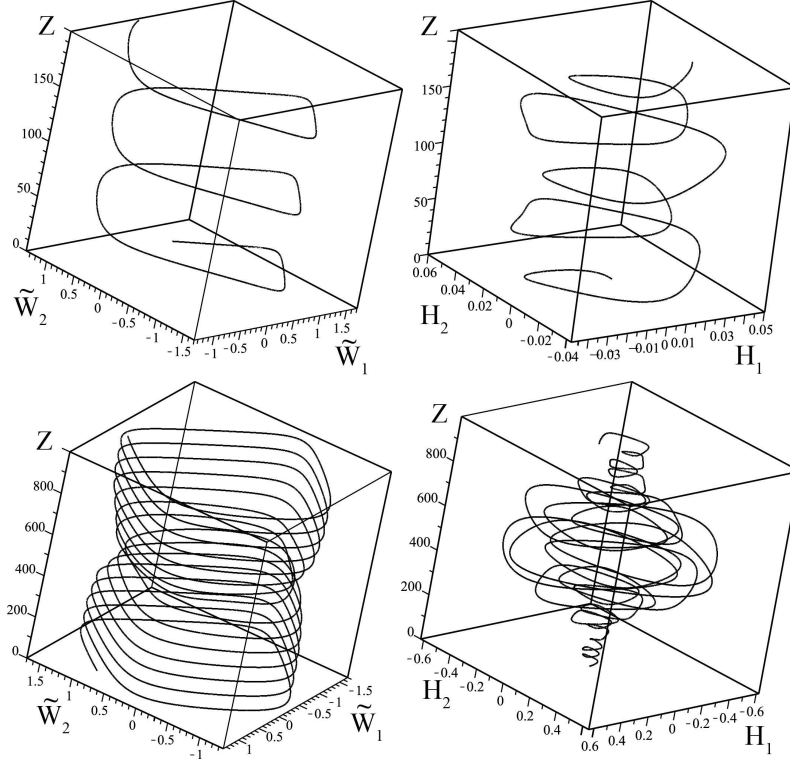


Figure 3: The upper part shows the velocity and magnetic field dependence on the height for the numerical solution of equations (44) - (47) with the initial conditions: $\widetilde{W}_1(0) = 0.8$, $\widetilde{W}_2(0) = 0.8$, $H_1(0) = 0.01$, $H_2(0) = 0.01$. This dependence corresponds to regular motion of the Poincare section shown on top of Fig. 2. Below a similar dependence for the numerical solution of equations ((44)) - ((47)) with the initial conditions: $\widetilde{W}_1(0) = 0.9$, $\widetilde{W}_2(0) = 0.9$, $H_1(0) = 0.01$, $H_2(0) = 0.01$. This chaotic dependence corresponds to Poincare sections in Fig.2 shown at the bottom.

stationary trajectories let us consider the Poincare section of the trajectories in the phase space. Fig.2 shows examples of the cross-sections obtained numerically for the dimensionless parameters $Q = Pm = Pr = 1$, $Ra = 2$ and the constants $C_1 = C_2 = 0.01$, $C_3 = C_4 = 0.001$. The upper part of Fig.2 demonstrates the Poincare section of regular trajectory for the velocity and magnetic fields. The structure of the chaotic layer, which belongs to the selected path is clearly visible on the lower section. The presence of such chaotic trajectories implies the existence of a stationary random structure of the velocity and magnetic fields. The chaotic change of velocity direction of magnetic fields according to the height Z is typical for these trajectories. Thus this set of equations has stationary chaotic solutions. Fig.3 shows the dependence of stationary large-scale fields on the height of Z , which was obtained numerically for the initial conditions, which correspond to the Poincare sections in Fig.2. These figures show also the appearance of stationary random solutions for magnetic and vortex fields. It should be noted that the structure of the magnetic field at the bottom of Fig.3 demonstrates with the increasing of height the intermittency structure. In the numerical solution of equations (44)-(47) occur chaotic structures observed with the increase of the initial velocity amplitudes. For small initial velocities and magnetic fields regular trajectories are typical. With increasing of velocity amplitude, above a certain critical value the chaotic trajectories appear. With increasing of the initial velocity the part of the space occupied by the chaotic trajectories points grows on the Poincare sections. Under these conditions, typical solutions become chaotic.

Let us now consider in more detail which nonlinear structure may occur in the convective electrically conductive turbulent medium, in some limit cases. Let us suppose that the generation of large-scale vortex disturbances in the convective medium is not yet in the stationary mode, but large-scale perturbations of the magnetic field are already reached their saturation on stationary level. Then the influence of the small amplitude of large-scale vortex disturbances $W_{1,2} \ll 1$ on the evolution of large-scale magnetic fields can be neglected. As a result, from the equations of the nonlinear dynamo (25) - (28), we obtain the equations for the evolution of the large-scale stationary magnetic field:

$$\frac{dH_1}{dZ} = -\frac{H_2(Q^2 H_2^4 + RaQH_2^2 + 4)}{(Q^2 H_2^4 + 4)(Q^2 H_2^4 + 2RaQH_2^2 + Ra^2 + 4)} + C_3 \quad (48)$$



Figure 4: On the left the phase portrait of Hamiltonian equations (51) at $C_3 = C_4 = 0$; on the right, the solution corresponding to the nonlinear wave.

$$\frac{dH_2}{dZ} = \frac{H_1(Q^2 H_1^4 + RaQH_1^2 + 4)}{(Q^2 H_1^4 + 4)(Q^2 H_1^4 + 2RaQH_1^2 + Ra^2 + 4)} + C_4 \quad (49)$$

Here, to simplify the calculation we use the Prandtl numbers: $Pr = Pm = 1$. Equation (48)-(49) can be written in the Hamiltonian form:

$$\frac{d\tilde{X}}{dt} = -\frac{d\mathcal{H}}{d\tilde{P}}, \quad \frac{d\tilde{P}}{dt} = \frac{d\mathcal{H}}{d\tilde{X}} \quad (50)$$

where we have introduced new variables $\tilde{X} = \sqrt{Q}H_1$, $\tilde{P} = \sqrt{Q}H_2$, $t = Z$. The Hamiltonian of the magnetic field \mathcal{H} is of the form:

$$\mathcal{H} = U(\tilde{P}) + U(\tilde{X}) - \tilde{C}_3 P + \tilde{C}_4 X + C_5 \quad (51)$$

where the function $U(y)$ is equal to:

$$U(y) = \frac{Ra}{4(Ra^2 + 16)} \ln \left(\frac{y^4 + 4}{y^4 + 4 + 2Ray^2 + Ra^2} \right) + \frac{2}{Ra^2 + 16} \left(\arctg \left(\frac{y^2}{2} \right) + \arctg \left(\frac{y^2 + Ra}{2} \right) \right) \quad (52)$$

The constant \tilde{C}_3 and \tilde{C}_4 , respectively, are: $\tilde{C}_3 = \sqrt{Q}C_3$, $\tilde{C}_4 = \sqrt{Q}C_4$. We examine now the types of stationary magnetic structures described by equations (50). First of all we consider in detail the limit case $Ra \rightarrow 0$. Then the Hamiltonian (51) takes the following form:

$$\mathcal{H} \rightarrow \mathcal{H}' = \frac{1}{4Q} \arctg \left(\frac{QH_1^2}{2} \right) + \frac{1}{4Q} \arctg \left(\frac{QH_2^2}{2} \right) + C_4 H_1 - C_3 H_2 + \tilde{C}_5 \quad (53)$$

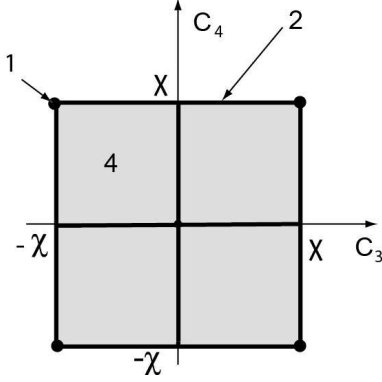


Figure 5: On the parameter plane (C_3, C_4) gray area shows the presence of four fixed points. For constants $(C_3 = 0, C_4 = 0)$, there is only one fixed point. The bold lines mark the values of the constants which correspond to the two fixed points on phase portrait. The vertices of the square area correspond to the values at which exist one fixed point. For values outside this area fixed points are absent.

The Hamiltonian equations (50) mean that in the phase space only fixed points of two types can be observed: elliptical and hyperbolic fixed points. At zero values of the constants $C_3 = C_4 = 0$ it is possible to construct the phase portrait of Hamiltonian (53) from which it is clear that there is only one elliptic point (see Fig.4). Around the elliptic points are observed the nonlinear waves only. The numerical stationary solution of equations (48)-(49) in the limit of $Ra \rightarrow 0$ corresponds to a nonlinear wave of finite amplitude and is shown in Fig.4. Calculating the maximum and minimum of $\left(\frac{H_{1,2}}{Q^2 H_{1,2}^4 + 4}\right)$ we find the area of the parameter (C_3, C_4) change defined by the inequalities:

$$-\chi < C_3 < \chi$$

$$-\chi < C_4 < \chi$$

where $\chi = \frac{3^{3/4}}{16} \sqrt{\frac{2}{Q}}$ depends on the choice of the parameter Q .

In Fig.5 the corresponding area is presented with the number of fixed points. In the vertices of square area of the parameters (C_3, C_4) is located a fixed point of the elliptical type. The type of fixed point is determined by the form of phase portrait of Hamiltonian (53) for the constants $C_3 =$

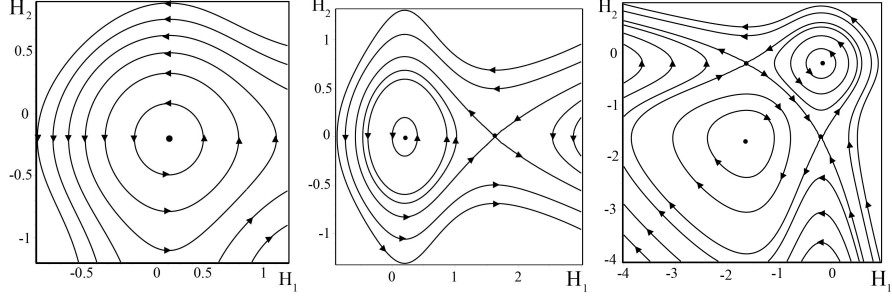


Figure 6: On the left the phase portrait for the fixed points located at vertices of the square area of parameters (C_3, C_4) ; the center-phase portrait for the fixed points on the boundary of the parameters (C_3, C_4) , fixed points when $(C_3 = 0, C_4 = \pm\chi)$; $(C_3 = \pm\chi, C_4 = 0)$; on the right the phase portrait for the fixed points inside the square area of parameters (C_3, C_4) .

$\pm\chi$ and $C_4 = \pm\chi$ (see Fig.6). On the boundary of (C_3, C_4) are two fixed points: the elliptical and hyperbolic one. Phase portrait for these points is shown in Fig.6. For constants $(C_3 = 0, C_4 = ([-\chi, 0[,]0, -\chi]))$, $C_3 = ([-\chi, 0[,]0, -\chi])$, $C_4 = 0$ there are also two fixed points: the elliptical and hyperbolic one. The phase portrait for this point is shown in Fig.6. At last, within each sector of the area (C_3, C_4) are four fixed points, two elliptical and two hyperbolic. Phase portraits for these points, are shown in Fig.6. Fig.7 shows a three-dimensional image of the limited stationary structures corresponding to the phase portraits presented in Fig.7. The left side of Fig.7 shows the numerical solution corresponding to a nonlinear wave that occurs in the vicinity of elliptic point in the phase space. The central part of Fig.7 shows the solitons solution, which correspond to the separatrix part going out and in the hyperbolic point. Finally, the right side of Fig.7 presents a solution for the kink, which corresponds to the part of the separatrix linking two hyperbolic points.

Let us now turn to the evolution of large-scale stationary magnetic field for the case when the Rayleigh number $Ra \rightarrow 2$. In this limit, the Hamiltonian (51) takes the following form:

$$\begin{aligned} \mathcal{H} \rightarrow \mathcal{H}' = & \frac{1}{40Q} \ln \left(\frac{Q^2 H_1^4 + 4}{Q^2 H_1^4 + 4QH_1^2 + 8} \right) + \frac{1}{40Q} \ln \left(\frac{Q^2 H_2^4 + 4}{Q^2 H_2^4 + 4QH_2^2 + 8} \right) + \\ & + \frac{1}{10Q} \arctg \left(\frac{4(QH_1^2 + 1)}{4 - Q^2 H_1^4 - 2QH_1^2} \right) + \frac{1}{10Q} \arctg \left(\frac{4(QH_2^2 + 1)}{4 - Q^2 H_2^4 - 2QH_2^2} \right) \end{aligned}$$

$$+ C_4 H_1 - C_3 H_2 + \tilde{C}_5 \quad (54)$$

At zero values of the constants $C_3 = C_4 = 0$ the phase portrait of Hamiltonian ((54)) is similar to the Fig.4. which implies the appearance of only one elliptic point in the phase space. In this case, all appearing stationary solutions coincide with the nonlinear waves. The stationary solution corresponding to the nonlinear finite amplitude wave are similar to the Fig.4.

As in the previous case it is easy to set the parameters for the area C_3, C_4 with a different number of fixed points. We notice that the region of existence of fixed points of the parameter plane (C_3, C_4) is determined by the inequalities:

$$\begin{aligned} -\tilde{\chi} &< C_3 < \tilde{\chi} \\ -\tilde{\chi} &< C_4 < \tilde{\chi} \end{aligned}$$

where $\tilde{\chi} = \max \left(\frac{H_1(Q^2 H_1^4 + Ra Q H_1^2 + 4)}{(Q^2 H_1^4 + 4)(Q^2 H_1^4 + 2Ra Q H_1^2 + Ra^2 + 4)} \right)$ when H_1 is changing. For fixed values of the parameters Ra, Q , this maximum can be easily calculated. From the results of these calculations, we obtain the similar phase portraits of the Hamiltonian (54) and numerical solutions of equations (48)-(49) at $Ra \rightarrow 2$ like in the previous case. Thus, comparing the growth rate of the vortex and magnetic disturbances at the initial stage of large-scale development of the instability, we considered the emergence of large-scale stationary magnetic structures. These structures are classified as stationary solutions of three types: nonlinear waves, solitons and kinks.

6 Conclusions

In this paper the closed system of nonlinear equations was obtained using the asymptotic method. These equations describe both linear and nonlinear increase stages of hydrodynamic flows and magnetic fields in a electroconducting medium. This system of equations allows to explain emergence and stabilization of large-scale magnetic fields of some astrophysical objects, stars, in particular. It is also interesting to use it to describe the generation of large-scale fields by convection in electrically conductive medium in the interior of planets. It should be noted that despite the asymptotic technique based on the presence of small-scale oscillations that is used, it is expected that the results can be applicable to the turbulent media. The turbulent case has the whole range of these small-scale oscillations. Qualitative evaluation

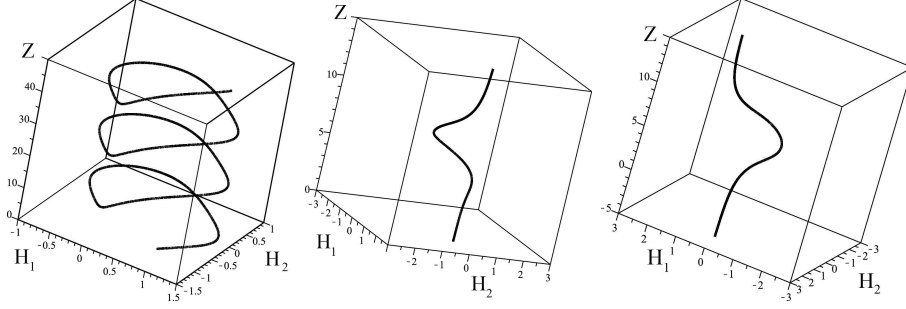


Figure 7: The numerical stationary solutions of equations (48) - (49) at $Ra \rightarrow 0$ in the form of nonlinear wave, soliton and kink. They correspond to the phase portraits, shown in Figure 4.

of the linear stage for the solar conditions [20] allow us to state a good coincidence of the resulting hydrodynamic structures characteristic scales and times with observation data [21]. This fact suggests that other stationary magneto-hydrodynamic structures, like soliton exist in the Sun photosphere.

Appendix I. Multiscale asymptotic developments

Let us find the algebraic structure of the asymptotic development in various orders of R , starting with the lowest. In order of R^{-3} there is only one equation:

$$\partial_i P_{-3} = 0 \Rightarrow P_{-3} = P_{-3}(X) \quad (55)$$

In order R^{-2} appears equation:

$$\partial_i P_{-2} = 0 \Rightarrow P_{-2} = P_{-2}(X) \quad (56)$$

Consequently, quantities P_{-3} and P_{-2} depend only on fast variables. In order R^{-1} , we obtain more complicated system of equations:

$$\begin{aligned} \partial_t W_{-1}^i + W_{-1}^k \partial_k W_{-1}^i = & -\partial_i P_{-1} - \nabla_i P_{-3} + \partial_k^2 W_{-1}^i + \\ & + \widetilde{R} a e_i T_{-1} + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \partial_m B_{-1}^l B_{-1}^k \end{aligned} \quad (57)$$

$$\partial_t B_{-1}^i - P m^{-1} \partial_k^2 B_{-1}^i = \varepsilon_{ijk} \varepsilon_{knp} \partial_j W_{-1}^n B_{-1}^p \quad (58)$$

$$\partial_t T_{-1} - Pr^{-1} \partial_k^2 T_{-1} = -W_{-1}^k \partial_k T_{-1} - W_{-1}^z \quad (59)$$

$$\partial_i W_{-1}^i = 0, \quad \partial_i B_{-1}^i = 0 \quad (60)$$

The averaging of equations (57)-(60) on the fast variables give the following secular equations:

$$-\nabla_i P_{-3} + \widetilde{R} a e_i T_{-1} = 0 \quad (61)$$

$$W_{-1}^z = 0 \quad (62)$$

In zero order in R we have the equations:

$$\begin{aligned} \partial_t v_0^i + W_{-1}^k \partial_k v_0^i + v_0^k \partial_k W_{-1}^i &= -\partial_i P_0 - \nabla_i P_{-2} + \\ + \partial_k^2 v_0^i + \widetilde{R} a e_i T_0 + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} (\partial_m B_{-1}^l B_0^k + \partial_m B_0^l B_{-1}^k) &+ F_0^i \end{aligned} \quad (63)$$

$$\partial_t B_0^i - P m^{-1} \partial_k^2 B_0^i = \varepsilon_{ijk} \varepsilon_{knp} (\partial_j W_{-1}^n B_0^p + \partial_j v_0^n B_{-1}^p) \quad (64)$$

$$\partial_t T_0 - Pr^{-1} \partial_k^2 T_0 = -W_{-1}^k \partial_k T_0 - \partial_k (v_0^k T_{-1}) - v_0^z \quad (65)$$

$$\partial_i v_0^i = 0, \quad \partial_i B_0^i = 0 \quad (66)$$

These equations give one secular equation:

$$\nabla P_{-2} = 0 \quad \Rightarrow \quad P_{-2} = \text{const} \quad (67)$$

Let us consider the equations of the first approximation R^1 :

$$\begin{aligned} \partial_t v_1^i + W_{-1}^k \partial_k v_1^i + v_0^k \partial_k v_0^i + v_1^k \partial_k W_{-1}^i + W_{-1}^k \nabla_k W_{-1}^i &= \\ = -\nabla_i P_{-1} - \partial_i (P_1 + \overline{P}_1) + \partial_k^2 v_1^i + 2 \partial_k \nabla_k W_{-1}^i + \widetilde{R} a e_i T_1 + \\ + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \partial_m B_{-1}^l B_1^k + \partial_m B_0^l B_0^k + \partial_m B_1^l B_{-1}^k + \nabla_m B_{-1}^l B_{-1}^k) \end{aligned} \quad (68)$$

$$\begin{aligned} \partial_t B_1^i - P m^{-1} \partial_k^2 B_1^i - P m^{-1} 2 \partial_k \nabla_k B_{-1}^i &= \\ = \varepsilon_{ijk} \varepsilon_{knp} (\partial_j W_{-1}^n B_1^p + \partial_j v_0^n B_0^p + \partial_j v_1^n B_{-1}^p + \nabla_j W_{-1}^n B_{-1}^p) \end{aligned} \quad (69)$$

$$\begin{aligned} \partial_t T_1 - Pr^{-1} \partial_k^2 T_1 - Pr^{-1} 2 \partial_k \nabla_k T_{-1} &= \\ = -W_{-1}^k \partial_k T_1 - W_{-1}^k \nabla_k T_{-1} - v_0^k \partial_k T_0 - v_1^k \partial_k T_{-1} - v_1^z \end{aligned} \quad (70)$$

$$\partial_i v_1^i + \nabla_i W_{-1}^i = 0, \quad \partial_i B_1^i + \nabla_i B_{-1}^i = 0 \quad (71)$$

The secular equations follow from this system of equations:

$$W_{-1}^k \nabla_k W_{-1}^i = -\nabla_i P_{-1} + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \nabla_m B_{-1}^l B_{-1}^k \quad (72)$$

$$\varepsilon_{ijk}\varepsilon_{knp}\nabla_j W_{-1}^n B_{-1}^p = 0 \quad (73)$$

$$W_{-1}^k \nabla_k T_{-1} = 0, \quad (74)$$

$$\nabla_i W_{-1}^i = 0, \quad \nabla_i B_{-1}^i = 0 \quad (75)$$

The secular equation (72)-(75) are satisfied by choosing the following geometry for the velocity and magnetic fields (Beltrami fields):

$$\vec{W}_{-1} = (W_x(Z), W_y(Z), 0),$$

$$\vec{B}_{-1} = (B_{-1}^x(Z), B_{-1}^y(Z), 0), \quad T_{-1} = T_{-1}(Z), \quad P_{-1} = \text{const} \quad (76)$$

In the second order R^2 , we obtain the equations:

$$\begin{aligned} \partial_t v_2^i + W_{-1}^k \partial_k v_2^i + v_0^k \partial_k v_1^i + W_{-1}^k \nabla_k v_0^i + v_0^k \nabla_k W_{-1}^i + v_1^k \partial_k v_0^i + v_2^k \partial_k W_{-1}^i = \\ = -\nabla_i P_2 - \nabla_i P_0 + \partial_k^2 v_2^i + 2\partial_k \nabla_k v_0^i + \widetilde{R} a e_i T_2 + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} (\partial_m B_{-1}^l B_2^k + \\ + \partial_m B_0^l B_1^k + \partial_m B_1^l B_0^k + \partial_m B_2^l B_{-1}^k + \nabla_m B_{-1}^l B_0^k + \nabla_m B_0^l B_{-1}^k) \end{aligned} \quad (77)$$

$$\begin{aligned} \partial_t B_2^i - P m^{-1} \partial_k^2 B_2^i - P m^{-1} 2\partial_k \nabla_k B_0^i = \varepsilon_{ijk} \varepsilon_{knp} (\partial_j W_{-1}^n B_2^p + \\ + \partial_j v_0^n B_1^p + \partial_j v_1^n B_0^p + \partial_j v_2^n B_{-1}^p + \nabla_j W_{-1}^n B_0^p + \nabla_j v_0^n B_{-1}^p) \end{aligned} \quad (78)$$

$$\begin{aligned} \partial_t T_2 - P r^{-1} \partial_k^2 T_2 - P r^{-1} 2\partial_k \nabla_k T_0 = -W_{-1}^k \partial_k T_2 - W_{-1}^k \nabla_k T_0 - \\ - v_0^k \partial_k T_1 - v_0^k \nabla_k T_{-1} - v_1^k \partial_k T_0 - v_2^k \partial_k T_{-1} - v_2^z \end{aligned} \quad (79)$$

$$\partial_i v_2^i + \nabla_i v_0^i = 0, \quad \partial_i B_2^i + \nabla_i B_0^i = 0 \quad (80)$$

It is easy to see that there are no secular terms in this order. Let us consider now the most important order R^3 . In this order we obtain the equations:

$$\begin{aligned} \partial_t v_3^i + \partial_T W_{-1}^i + W_{-1}^k \partial_k v_3^i + v_0^k \partial_k v_2^i + \\ + W_{-1}^k \nabla_k v_1^i + v_0^k \nabla_k v_0^i + v_1^k \partial_k v_1^i + v_2^k \partial_k v_0^i + v_1^k \nabla_k W_{-1}^i + v_3^k \partial_k W_{-1}^i = \\ = -\partial_i P_3 - \nabla_i (P_1 + \overline{P}_1) + \partial_k^2 v_3^i + 2\partial_k \nabla_k v_1^i + \Delta W_{-1}^i + \widetilde{R} a e_i T_3 + \\ + \widetilde{Q} \varepsilon_{ijk} \varepsilon_{jml} (\partial_m B_{-1}^l B_3^k + \partial_m B_0^l B_2^k + \partial_m B_1^l B_1^k + \\ + \partial_m B_2^l B_0^k + \nabla_m B_{-1}^l B_1^k + \nabla_m B_0^l B_0^k), \end{aligned} \quad (81)$$

$$\begin{aligned} \partial_t B_3^i + \partial_T B_{-1}^i - P m^{-1} \partial_k^2 B_3^i - P m^{-1} 2\partial_k \nabla_k B_1^i - P m^{-1} \Delta B_{-1}^i = \\ = \varepsilon_{ijk} \varepsilon_{knp} (\partial_j W_{-1}^n B_3^p + \partial_j v_0^n B_2^p + \partial_j v_1^n B_1^p + \partial_j v_2^n B_0^p + \end{aligned}$$

$$+ \nabla_j W_{-1}^n B_1^p + \nabla_j v_0^n B_0^p), \quad (82)$$

$$\begin{aligned} \partial_t T_3 + \partial_T T_{-1} - Pr^{-1} \partial_k^2 T_3 - Pr^{-1} 2 \partial_k \nabla_k T_1 - Pr^{-1} \Delta T_{-1} = \\ = -W_{-1}^k \partial_k T_3 - W_{-1}^k \nabla_k T_1 - v_0^k \partial_k T_2 - v_0^k \nabla_k T_0 - v_1^k \partial_k T_1 - \\ - v_1^k \nabla_k T_{-1} - v_2^k \partial_k T_0 - v_3^k \partial_k T_{-1} - v_3^z \end{aligned} \quad (83)$$

$$\partial_i v_3^i + \nabla_i v_1^i = 0, \quad \partial_i B_3^i + \nabla_i B_1^i = 0 \quad (84)$$

After averaging this system of equations over fast variables, we obtain the main system of secular equations to describe the evolution of large-scale perturbations:

$$\partial_t W_{-1}^i - \Delta W_{-1}^i + \nabla_k \left(\overline{v_0^k v_0^i} \right) = -\nabla_i \overline{P}_1 + \tilde{Q} \varepsilon_{ijk} \varepsilon_{jml} \left(\overline{\nabla_m B_0^l B_0^k} \right) \quad (85)$$

$$\partial_t B_{-1}^i - Pm^{-1} \Delta B_{-1}^i = \varepsilon_{ijk} \varepsilon_{knp} \nabla_j \left(\overline{v_0^n B_0^p} \right) \quad (86)$$

$$\partial_t T_{-1} - Pr^{-1} \Delta T_{-1} = -\nabla_k \left(\overline{v_0^k T_0} \right) \quad (87)$$

Using the well-known tensor identities:

$$\varepsilon_{ijk} \varepsilon_{jml} = \delta_{km} \delta_{il} - \delta_{im} \delta_{kl},$$

$$\varepsilon_{ijk} \varepsilon_{knp} = \delta_{in} \delta_{jp} - \delta_{ip} \delta_{jn}$$

and introducing for convenience designations $\vec{W} = \vec{W}_{-1}$, $\vec{H} = \vec{B}_{-1}$, $\Theta = T_{-1}$, we write equations (85)-(87) in the following form:

$$\partial_T W_i - \Delta W_i + \nabla_k \left(\overline{v_0^k v_0^i} \right) = -\nabla_i \overline{P} + \tilde{Q} \left(\nabla_k \left(\overline{B_0^i B_0^k} \right) - \frac{\nabla_i}{2} \left(\overline{B_0^k} \right)^2 \right) \quad (88)$$

$$\partial_T H_i - Pm^{-1} \Delta H_i = \nabla_j \left(\overline{v_0^i B_0^j} \right) - \nabla_n \left(\overline{v_0^n B_0^i} \right) \quad (89)$$

$$\partial_T \Theta - Pr^{-1} \Delta \Theta + \nabla_k \left(\overline{v_0^k T_0} \right) = 0 \quad (90)$$

Appendix II. Small-scale fields

In Appendix I, we obtain the equations in the zero order in R , which can be written using a more compact denagnations for operators:

$$\begin{aligned}\hat{D}_W &= \partial_t - \partial^2 + W_{-1}^k \partial_k, \\ \hat{D}_H &= \partial_t - Pm^{-1} \partial^2 + W_{-1}^k \partial_k, \quad \hat{D}_\theta = \partial_t - Pr^{-1} \partial^2 + W_{-1}^k \partial_k,\end{aligned}\tag{91}$$

Then the set of equations (63)-(66) takes the form:

$$\hat{D}_W v_0^i = -\partial_i P_0 + \widetilde{R} a e_i T_0 + \widetilde{Q} H_k (\partial_k B_0^i - \partial_i B_0^k) + F_0^i,\tag{92}$$

$$\hat{D}_H B_0^i = H_p \partial_p v_0^i,\tag{93}$$

$$\hat{D}_\theta T_0 = -e_k v_0^k,\tag{94}$$

$$\partial_i v_0^i = \partial_k B_0^k = 0,\tag{95}$$

It is not difficult to find expressions for the small-scale field \vec{B}_0 and T_0 :

$$B_0^i = \frac{H_p \partial_p v_0^i}{\hat{D}_H}, \quad T_0 = -\frac{e_k v_0^k}{\hat{D}_\theta}\tag{96}$$

Now we substitute (96) in the equation (92), then differentiate the resulting expression for ∂_i , using the conditions of solenoidal fields (95). We obtain an expression for the pressure P_0 :

$$P_0 = -\frac{\widetilde{R} a e_i e_k \partial_i v_0^k}{\partial^2 \hat{D}_\theta} - \frac{\widetilde{Q}}{\partial^2 \hat{D}_H} (H_p \partial_p) (H_k \partial^2 v_0^k)\tag{97}$$

Eliminating P_0 from (92) we obtain the equation for v_0^k :

$$\left(\delta_{ik} + \frac{\widetilde{R} a}{\hat{q} \hat{D}_W \hat{D}_\theta} \hat{P}_{ip} e_p e_k \right) v_0^k = \frac{F_0^i}{\hat{q} \hat{D}_W},\tag{98}$$

where $\hat{P}_{ip} = \delta_{ip} - \frac{\partial_i \partial_p}{\partial^2}$ is the projection operator,

$$\hat{q} = 1 - \frac{\widetilde{Q} (H_k \partial_k)^2}{\hat{D}_W \hat{D}_H}.$$

Let us rewrite (98) in the most compact form:

$$\hat{L}_{ik}v_0^k = \frac{F_0^i}{\hat{q}\hat{D}_W}, \quad (99)$$

where the designation for the operator

$$\hat{L}_{ik} = \delta_{ik} + \frac{\widetilde{Ra}\hat{P}_{ip}e_pe_k}{\hat{q}\hat{D}_W\hat{D}_\theta}. \quad (100)$$

From (99) we can find directly through the inverse operator \hat{L}_{kj}^{-1} a velocity field v_0^k , i.e.

$$v_0^k = L_{kj}^{-1} \frac{F_0^j}{\hat{q}\hat{D}_W}, \quad (101)$$

where \hat{L}_{kj}^{-1} has the property $\hat{L}_{ik}\hat{L}_{kj}^{-1} = \delta_{ij}$:

$$\hat{L}_{kj}^{-1} = \delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}\hat{D}_W\hat{D}_\theta + \widetilde{Ra}\hat{P}_{qp}e_qe_p} \quad (102)$$

The expression for the small-scale fluctuations of velocity v_0^k takes the form:

$$v_0^k = \left[\delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}\hat{D}_W\hat{D}_\theta + \widetilde{Ra}\hat{P}_{qp}e_qe_p} \right] \frac{F_0^j}{\hat{q}\hat{D}_W} \quad (103)$$

Small-scale fluctuations of temperature T_0 expressed in terms of $\vec{v}_0(\vec{F}_0)$:

$$T_0 = - \left[1 - \frac{\widetilde{Ra}\hat{P}_{kn}e_ke_n}{\hat{q}\hat{D}_W\hat{D}_\theta + \widetilde{Ra}\hat{P}_{qp}e_qe_p} \right] \frac{(\vec{e}\vec{F}_0)}{\hat{q}\hat{D}_W\hat{D}_\theta} \quad (104)$$

Equations (103) - (104) in the limit $Pr = 1$ and $\sigma = 0$ (non-electroconductive medium) fully agree with the results of [16], [17]. Next, we need to know the explicit form for small-scale fluctuations of magnetic fields \vec{B}_0 :

$$B_0^k = \left[\delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}\hat{D}_W\hat{D}_\theta + \widetilde{Ra}\hat{P}_{qp}e_qe_p} \right] \frac{H_p\partial_p F_0^j}{\hat{q}\hat{D}_W D_H} \quad (105)$$

For further calculations of correlators we need to set explicitly the external helical force \vec{F}_0 in a deterministic form:

$$\vec{F}_0 = f_0 \left[\vec{i} \cos \varphi_2 + \vec{j} \sin \varphi_1 + \vec{k}(\cos \varphi_1 + \cos \varphi_2) \right] \quad (106)$$

where $\varphi_1 = \vec{\kappa}_1 \vec{x} - \omega_0 t$, $\varphi_2 = \vec{\kappa}_2 \vec{x} - \omega_0 t$, $\kappa_1 = \kappa_0 (1, 0, 0)$, $\kappa_2 = \kappa_0 (0, 1, 0)$. Then the helicity of the external force is equal to:

$$\vec{F}_0 \text{rot} \vec{F}_0 = \kappa_0 \vec{F}_0^2 \neq 0 \quad (107)$$

It is convenient to rewrite the equation (106) in the complex form:

$$\vec{F}_0 = \vec{A} e^{i\varphi_1} + \vec{A}^* e^{-i\varphi_1} + \vec{B} e^{i\varphi_2} + \vec{B}^* e^{-i\varphi_2} \quad (108)$$

where the vectors \vec{A} and \vec{B} are respectively:

$$\vec{A} = \frac{f_0}{2} (\vec{k} - i\vec{j}), \vec{A}^* = \frac{f_0}{2} (\vec{k} + i\vec{j}), \vec{B} = \frac{f_0}{2} (\vec{i} - i\vec{k}), \vec{B}^* = \frac{f_0}{2} (\vec{i} + i\vec{k}) \quad (109)$$

Action of the operators \hat{q} , \hat{D}_W , \hat{D}_H , \hat{D}_θ on their eigenfunctions $\exp(i\omega t + i\vec{\kappa} \vec{x})$ obviously has the form:

$$\begin{aligned} \hat{q}(\omega, \vec{\kappa}) \exp(i\vec{\kappa} \vec{x} + i\omega t), \quad \hat{D}_W(\omega, \vec{\kappa}) \exp(i\vec{\kappa} \vec{x} + i\omega t), \\ \hat{D}_H(\omega, \vec{\kappa}) \exp(i\vec{\kappa} \vec{x} + i\omega t), \quad \hat{D}_\theta(\omega, \vec{\kappa}) \exp(i\vec{\kappa} \vec{x} + i\omega t), \end{aligned}$$

where $\hat{q}(\omega, \vec{\kappa})$, $\hat{D}_W(\omega, \vec{\kappa})$, $\hat{D}_H(\omega, \vec{\kappa})$, $\hat{D}_\theta(\omega, \vec{\kappa})$ have the form:

$$\hat{q}(\omega, \vec{\kappa}) = 1 + \frac{\tilde{Q}(\vec{\kappa} \vec{H})^2}{\hat{D}_W(\omega, \vec{\kappa}) \hat{D}_H(\omega, \vec{\kappa})} \quad (110)$$

$$\begin{aligned} \hat{D}_W(\omega, \vec{\kappa}) &= i \left(\omega + \vec{\kappa} \vec{W} \right) + \kappa^2 \\ \hat{D}_H(\omega, \vec{\kappa}) &= i \left(\omega + \vec{\kappa} \vec{W} \right) + \kappa^2 P m^{-1} \\ \hat{D}_\theta(\omega, \vec{\kappa}) &= i \left(\omega + \vec{\kappa} \vec{W} \right) + \kappa^2 P r^{-1} \end{aligned}$$

We write down the number of useful relations, assuming for simplicity that $\kappa_0 = 1$ and $\omega_0 = 1$:

$$\begin{aligned} \hat{D}_W(\omega_0, -\vec{\kappa}_1) &= i(1 - W_1) + 1 = \hat{D}_{W_1}, \hat{D}_W(\omega_0, -\vec{\kappa}_1) = \hat{D}_{W_1}^*, \\ \hat{D}_H(\omega_0, -\vec{\kappa}_1) &= i(1 - W_1) + P m^{-1} = \hat{D}_{H_1}, \hat{D}_H(\omega_0, -\vec{\kappa}_1) = \hat{D}_{H_1}^*, \\ \hat{D}_\theta(\omega_0, -\vec{\kappa}_1) &= i(1 - W_1) + P r^{-1} = \hat{D}_{\theta_1}, \hat{D}_\theta(\omega_0, -\vec{\kappa}_1) = \hat{D}_{\theta_1}^* \end{aligned} \quad (111)$$

$$\hat{q}(\omega_0, -\vec{\kappa}_1) = 1 + \frac{\tilde{Q}H_1^2}{(i(1-W_1)+1)(i(1-W_1)+Pm^{-1})} = \hat{q}_1$$

$$\hat{q}(-\omega_0, \vec{\kappa}_1) = \hat{q}_1^*, \quad H_1 = H_x, \quad W_1 = W_x$$

Similarly for the vector $\vec{\kappa}_2$ we get:

$$\hat{D}_W(\omega_0, -\vec{\kappa}_2) = i(1-W_2) + 1 = \hat{D}_{W_2}, \quad \hat{D}_W(\omega_0, -\vec{\kappa}_2) = \hat{D}_{W_2}^*,$$

$$\hat{D}_H(\omega_0, -\vec{\kappa}_2) = i(1-W_2) + Pm^{-1} = \hat{D}_{H_2}, \quad \hat{D}_H(\omega_0, -\vec{\kappa}_2) = \hat{D}_{H_2}^* \quad (112)$$

$$\hat{D}_\theta(\omega_0, -\vec{\kappa}_2) = i(1-W_2) + Pr^{-1} = \hat{D}_{\theta_2}, \quad \hat{D}_\theta(\omega_0, -\vec{\kappa}_2) = \hat{D}_{\theta_2}^*$$

$$\hat{q}(\omega_0, -\vec{\kappa}_2) = 1 + \frac{\tilde{Q}H_2^2}{(i(1-W_2)+1)(i(1-W_2)+Pm^{-1})} = \hat{q}_2$$

$$\hat{q}(-\omega_0, \vec{\kappa}_2) = \hat{q}_2^*, \quad H_2 = H_y, \quad W_2 = W_y$$

According to the definition of external force \vec{F}_0 (108), the small-scale fields $\vec{v}_0, \vec{B}_0, T_0$ determined by formulas (103)-(105), each of them consist of four terms:

$$v_0^k = v_{01}^k + v_{02}^k + v_{03}^k + v_{04}^k;$$

$$B_0^i = B_{01}^i + B_{02}^i + B_{03}^i + B_{04}^i; \quad T_0 = T_{01} + T_{02} + T_{03} + T_{04}; \quad (113)$$

where $v_{02}^k = (v_{01}^k)^*, v_{04}^k = (v_{03}^k)^*, B_{02}^k = (B_{01}^k)^*, B_{04}^k = (B_{03}^k)^*, T_{02} = (T_{01})^*, T_{04} = (T_{03})^*$

$$v_{01}^k = e^{i\varphi_1} \left[\delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}_1^*\hat{D}_{W_1}^*\hat{D}_{\theta_1}^* + \widetilde{Ra}} \right] \frac{A_j}{\hat{q}_1^*\hat{D}_{W_1}^*} \quad (114)$$

$$v_{03}^k = e^{i\varphi_2} \left[\delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}_2^*\hat{D}_{W_2}^*\hat{D}_{\theta_2}^* + \widetilde{Ra}} \right] \frac{B_j}{\hat{q}_2^*\hat{D}_{W_2}^*} \quad (115)$$

$$T_{01} = -e^{i\varphi_1} \left[1 - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_k}{\hat{q}_1^*\hat{D}_{W_1}^*\hat{D}_{\theta_1}^* + \widetilde{Ra}} \right] \frac{(\vec{e}\vec{A})}{\hat{q}_1^*\hat{D}_{W_1}^*\hat{D}_{\theta_1}^*} \quad (116)$$

$$T_{03} = -e^{i\varphi_2} \left[1 - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_k}{\hat{q}_2^*\hat{D}_{W_2}^*\hat{D}_{\theta_2}^* + \widetilde{Ra}} \right] \frac{(\vec{e}\vec{B})}{\hat{q}_2^*\hat{D}_{W_2}^*\hat{D}_{\theta_2}^*} \quad (117)$$

$$B_{01}^k = e^{i\varphi_1} \left[\delta_{kj} - \frac{\widetilde{Ra}\hat{P}_{kn}e_ne_j}{\hat{q}_1^*\hat{D}_{W_1}^*\hat{D}_{\theta_1}^* + \widetilde{Ra}} \right] \frac{iH_1A_j}{\hat{q}_1^*\hat{D}_{W_1}^*\hat{D}_{H_1}^*} \quad (118)$$

$$B_{03}^k = e^{i\varphi_2} \left[\delta_{kj} - \frac{\widetilde{Ra} \hat{P}_{kn} e_n e_j}{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \widetilde{Ra}} \right] \frac{i H_2 B_j}{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{H_2}^*} \quad (119)$$

Here we take into account that $\hat{P}_{qs}(\kappa_1) e_q e_s = \hat{P}_{qs}(\kappa_2) e_q e_s = 1$ because $\vec{\kappa}_1, \vec{\kappa}_2 \perp \vec{e}$.

Appendix III. Reynolds stress, Maxwell stress and turbulent e.m.f.

Let us start with the calculation of Reynolds stresses $\overline{v_0^k v_0^i} = T_{(1)}^{ki} + T_{(2)}^{ki}$. We need the equations (114)-(115) and type of the external helical force (108)-(109). For simplicity, we assume that the dimensionless amplitude of the external helical force $f_0 = 1$. Then we have:

$$T_{(1)}^{ki} = \frac{1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2} \left\{ (A_k A_i^* + A_i A_k^*) - \hat{a} A_z^* (e_i A_k + e_k A_i) - \right. \\ \left. - \hat{a}^* A_z (e_i A_k^* + e_k A_i^*) + 2|\hat{a}|^2 |A_z|^2 e_i e_k \right\}, \quad (120)$$

Where are introduced the following designations:

$$|\hat{q}_1|^2 = \hat{q}_1 \hat{q}_1^* = \hat{q}_1^* \hat{q}_1 = \frac{(Pm^{-1} - (1 - W_1)^2 + \widetilde{Q} H_1^2)^2 + (1 - W_1) (1 + Pm^{-1})^2}{(1 + (1 - W_1)^2) (Pm^{-2} + (1 - W_1)^2)};$$

$$|\hat{D}_{W_1}|^2 = \hat{D}_{W_1}^* \hat{D}_{W_1} = 1 + (1 - W_1)^2 \quad (121)$$

$$\hat{a}^* = \frac{\widetilde{Ra}}{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* + \widetilde{Ra}}, \hat{a} = \frac{\widetilde{Ra}}{\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}}, |\hat{a}|^2 = \hat{a} \hat{a}^*, |A_z|^2 = A_z^* A_z$$

The expression for the $T_{(2)}^{ki}$ has the similar form:

$$T_{(2)}^{ki} = \frac{1}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2} \left\{ (B_k B_i^* + B_i B_k^*) - \hat{b} B_z^* (e_i B_k + e_k B_i) - \right. \\ \left. - \hat{b}^* B_z (e_i B_k^* + e_k B_i^*) + 2|\hat{b}|^2 |B_z|^2 e_i e_k \right\}, \quad (122)$$

Here

$$|\hat{q}_2|^2 = \hat{q}_2 \hat{q}_2^* = \hat{q}_2^* \hat{q}_2 = \frac{(Pm^{-1} - (1 - W_2)^2 + \tilde{Q}H_2^2)^2 + (1 - W_2)(1 + Pm^{-1})^2}{(1 + (1 - W_2)^2)(Pm^{-2} + (1 - W_2)^2)} \quad (123)$$

$$\left| \hat{D}_{W_2} \right|^2 = \hat{D}_{W_2}^* \hat{D}_{W_2} = 1 + (1 - W_2)^2;$$

$$\hat{b}^* = \frac{\widetilde{Ra}}{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \widetilde{Ra}}, \hat{b} = \frac{\widetilde{Ra}}{\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}}, \left| \hat{b} \right|^2 = \hat{b} \hat{b}^*, |B_z|^2 = B_z^* B_z \quad (124)$$

Equations (120)-(124), in the limit of non electroconductive medium ($\sigma = 0$) and $Pr = 1$, were obtained in [16], [17]. Correlators for magnetic fields (Maxwell stress) $S_{(1)}^{ik} + S_{(2)}^{ik}$ can be found using (118)-(119):

$$S_{(1)}^{ki} = \frac{H_1^2}{|\hat{q}_1|^2 \left| \hat{D}_{W_1} \right|^2 \left| \hat{D}_{H_1} \right|^2} \left\{ (A_i A_k^* + A_k A_i^*) - \hat{a} A_z^* (e_k A_i + e_i A_k) - \right. \\ \left. - \hat{a}^* A_z (e_i A_k^* + e_k A_i^*) + 2|\hat{a}|^2 |A_z|^2 e_i e_k \right\} \quad (125)$$

where $\left| \hat{D}_{H_1} \right|^2 = \hat{D}_{H_1}^* \hat{D}_{H_1} = Pm^{-2} + (1 - W_1)^2$;

$$S_{(2)}^{ki} = \frac{H_2^2}{|\hat{q}_2|^2 \left| \hat{D}_{W_2} \right|^2 \left| \hat{D}_{H_2} \right|^2} \left\{ (B_i B_k^* + B_k B_i^*) - \hat{b} B_z^* (e_k B_i + e_i B_k) - \right. \\ \left. - \hat{b}^* B_z (e_i B_k^* + e_k B_i^*) + 2|\hat{b}|^2 |B_z|^2 e_i e_k \right\} \quad (126)$$

where $\left| \hat{D}_{H_2} \right|^2 = \hat{D}_{H_2}^* \hat{D}_{H_2} = Pm^{-2} + (1 - W_2)^2$. Using (114)-(115) and (118)-(119), we obtain expressions for the mixed correlator $G_{(1)}^{ik} + G_{(2)}^{ik}$:

$$G_{(1)}^{ik} = \frac{i P m^{-1} H_1}{|\hat{q}_1|^2 \left| \hat{D}_{W_1} \right|^2 \left| \hat{D}_{H_1} \right|^2} \times \\ \times \left\{ (A_j A_i^* - A_i A_j^*) - \hat{a} A_z^* (e_i A_j - e_j A_i) - \hat{a}^* A_z (e_j A_i^* - e_i A_j^*) \right\} -$$

$$\begin{aligned}
& -\frac{(1-W_1)H_1}{|\hat{q}_1|^2|\hat{D}_{W_1}|^2|\hat{D}_{H_1}|^2} \left\{ (A_j A_i^* + A_i A_j^*) - \hat{a} A_z^* (e_i A_j + e_j A_i) - \right. \\
& \quad \left. - \hat{a}^* A_z (e_j A_i^* + e_i A_j^*) + 2|\hat{a}|^2 |A_z|^2 e_i e_j \right\} \quad (127)
\end{aligned}$$

$$\begin{aligned}
& G_{(2)}^{ij} = \frac{i P m^{-1} H_2}{|\hat{q}_2|^2|\hat{D}_{W_2}|^2|\hat{D}_{H_2}|^2} \times \\
& \times \left\{ (B_j B_i^* - B_i B_j^*) - \hat{b} B_z^* (e_i B_j - e_j B_i) - \hat{b}^* B_z (e_j B_i^* - e_i B_j^*) \right\} - \\
& -\frac{(1-W_2)H_2}{|\hat{q}_2|^2|\hat{D}_{W_2}|^2|\hat{D}_{H_2}|^2} \left\{ (B_j B_i^* + B_i B_j^*) - \hat{b} B_z^* (e_i B_j + e_j B_i) - \right. \\
& \quad \left. - \hat{b}^* B_z (e_j B_i^* + e_i B_j^*) + 2|\hat{b}|^2 |B_z|^2 e_i e_j \right\} \quad (128)
\end{aligned}$$

By simple replacement of indices $i \rightarrow n$, $j \rightarrow i$ we obtain the expressions for $G_{(1)}^{ni}$ and $G_{(2)}^{ni}$. Since we are interested in the evolution of large-scale fields \vec{W} and \vec{H} , and taking into account the geometry of the problem (20), we need to know the following Reynolds stress components:

$$\begin{aligned}
& T_{(1)}^{31} + T_{(2)}^{31}; \quad T_{(1)}^{32} + T_{(2)}^{32}; \quad S_{(1)}^{31} + S_{(2)}^{31}; \quad S_{(1)}^{32} + S_{(2)}^{32}; \\
& G_{(1)}^{13} + G_{(2)}^{13}; \quad G_{(1)}^{31} + G_{(2)}^{31}; \quad G_{(1)}^{23} + G_{(2)}^{23}; \quad G_{(1)}^{32} + G_{(2)}^{32} \quad (129)
\end{aligned}$$

Using the equation (120) we can find the expression for $T_{(1)}^{31}$, while putting $k = 3$, $i = 1$:

$$\begin{aligned}
T_{(1)}^{31} &= \frac{1}{|\hat{q}_1|^2|\hat{D}_{W_1}|^2} \left\{ (A_3 A_1^* + A_1 A_3^*) - \hat{a} A_z^* (e_1 A_3 + e_3 A_1) - \right. \\
& \quad \left. - \hat{a}^* A_z (e_1 A_3^* + e_3 A_1^*) + 2|\hat{a}|^2 |A_z|^2 e_1 e_3 \right\} \quad (130)
\end{aligned}$$

since $e_1 = 0$ and $A_1 = A_1^* = 0$, $A_z = A_3$, $T_{(1)}^{31} = 0$. In a similar way is calculated $T_{(2)}^{31}$ using (122), where indices $k = 3$ and $i = 1$, i.e.

$$T_{(2)}^{31} = \frac{1}{|\hat{q}_2|^2|\hat{D}_{W_2}|^2} \left\{ (B_3 B_1^* + B_1 B_3^*) - \hat{b} B_z^* (e_1 B_3 + e_3 B_1) - \right.$$

$$\begin{aligned}
& -\hat{b}^* B_z (e_1 B_3^* + e_3 B_1^*) + 2|\hat{b}|^2 |B_z|^2 e_1 e_3 \Big\} \\
& B_z = B_3
\end{aligned} \tag{131}$$

Considering, that $B_3 B_1^* + B_1 B_3^* = 0$ and $e_1 = 0$, the equation (131) is simplified:

$$T_{(2)}^{31} = \frac{i}{4|\hat{q}_2|^2 |\hat{D}_{W_2}|^2} (\hat{b}^* - \hat{b}) \tag{132}$$

or after substitution of (124), we have:

$$T_{(2)}^{31} = -\frac{i}{4} \frac{\widetilde{Ra}}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2} \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* - \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2}}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}|^2} \right\} \tag{133}$$

Taking in formulas (120) and (122) indices k and i equal respectively $k = 3$ and $i = 2$, we can obtain the expression for the $T_{(1)}^{31}$ and $T_{(2)}^{31}$:

$$\begin{aligned}
T_{(1)}^{32} &= \frac{1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2} \{ (A_3 A_2^* + A_2 A_3^*) - \hat{a} A_z^* (e_2 A_3 + e_3 A_2) - \\
& - \hat{a}^* A_z (e_2 A_3^* + e_3 A_2^*) + 2|\hat{a}|^2 |A_z|^2 e_2 e_3 \}
\end{aligned} \tag{134}$$

given the $e_2 = 0$, $A_3 A_2^* + A_2 A_3^* = 0$ and equation (121), the form of the expression (134) becomes simpler:

$$T_{(1)}^{32} = \frac{i}{4} \frac{\widetilde{Ra}}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* - \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} \right\} \tag{135}$$

Component $T_{(2)}^{32}$ is zero due to the fact that the $e_2 = 0$ and $B_2 = B_2^* = 0$:

$$\begin{aligned}
T_{(2)}^{32} &= \frac{1}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2} \left\{ (B_3 B_2^* + B_2 B_3^*) - \hat{b} B_z^* (e_2 B_3 + e_3 B_2) - \right. \\
& \left. - \hat{b}^* B_z (e_2 B_3^* + e_3 B_2^*) + 2|\hat{b}|^2 |B_z|^2 e_2 e_3 \right\}
\end{aligned} \tag{136}$$

In the equations (125) and (126) we take indices k and i equal to $k = 3$, $i = 1$. Then the expressions can be found for the components $S_{(1)}^{31}$ and $S_{(2)}^{31}$, respectively:

$$S_{(1)}^{31} = \frac{H_1^2}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \{ (A_3 A_1^* + A_1 A_3^*) - \hat{a} A_z^* (e_k A_i + e_i A_k) - \\ - \hat{a}^* A_3 (e_1 A_3^* + e_3 A_1^*) + 2|\hat{a}|^2 |A_3|^2 e_1 e_3 \} = 0 \quad (137)$$

i.e. $e_1 = 0$ and $A_1 = A_1^* = 0$;

$$S_{(2)}^{31} = \frac{H_2^2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ (B_3 B_1^* + B_1 B_3^*) - \hat{b} B_3^* (e_1 B_3 + e_3 B_1) - \right. \\ \left. - \hat{b}^* B_3 (e_3 B_1^* + e_1 B_3^*) + 2|\hat{b}|^2 |B_z|^2 e_3 e_1 \right\} \quad (138)$$

We take into account that $B_3 B_1^* + B_1 B_3^* = 0$ and $e_1 = 0$, then (138) takes the form:

$$S_{(2)}^{31} = \frac{i}{4} \frac{H_2^2 \widetilde{Ra}}{|\hat{q}_2|^2 |\hat{D}_{H_2}|^2 |\hat{D}_{W_2}|^2} \left\{ \frac{\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} - \hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^*}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}|^2} \right\} \quad (139)$$

From (125) and (126) we have the equations for the components of $S_{(1)}^{32}$ and $S_{(2)}^{32}$, assuming that $k = 3$, $i = 2$. Then

$$S_{(1)}^{32} = \frac{H_1^2}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \{ (A_3 A_2^* + A_2 A_3^*) - \hat{a} A_3^* (e_2 A_3 + e_3 A_2) - \\ - \hat{a}^* A_3 (e_3 A_2^* + e_2 A_3^*) + 2|\hat{a}|^2 |A_3|^2 e_3 e_2 \}$$

here $A_3 A_2^* + A_2 A_3^* = 0$, $e_2 = 0$. Then $S_{(1)}^{32}$ has the form:

$$S_{(1)}^{32} = \frac{i}{4} \frac{H_1^2 \widetilde{Ra}}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* - \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} \right\} \quad (140)$$

Component $S_{(2)}^{32}$ is equal to zero, i.e. $E_2 = 0$ and $B_2 = B_2^* = 0$:

$$S_{(2)}^{32} = \frac{H_2^2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ (B_3 B_2^* + B_2 B_3^*) - \hat{b} B_3^* (e_2 B_3 + e_3 B_1) - \right. \\ \left. - \hat{b}^* B_3 (e_3 B_2^* + e_2 B_3^*) + 2 |\hat{b}|^2 |B_3|^2 e_3 e_2 \right\} = 0 \quad (141)$$

Further, according to the equations (127) and (128) and replacing indices i and j by $i = 1$ and $j = 3$; $i = 3$ and $j = 1$; $i = 2$ and $j = 3$; $i = 3$ and $j = 2$, respectively, we get:

$$G_{(1)}^{13} = \frac{i P m^{-1} H_1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \times \\ \times \{ (A_3 A_1^* - A_1 A_3^*) - \hat{a} A_3^* (e_1 A_3 - e_3 A_1) - \hat{a}^* A_3 (e_3 A_1^* - e_1 A_3^*) \} - \\ - \frac{(1 - W_1) H_1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \{ (A_3 A_1^* + A_1 A_3^*) - \hat{a} A_3^* (e_1 A_3 + e_3 A_1) - \\ - \hat{a}^* A_3 (e_3 A_1^* + e_1 A_3^*) + 2 |\hat{a}|^2 |A_3|^2 e_1 e_3 \} \quad (142)$$

Since $e_1 = 0$ and $A_1 = A_1^* = 0$, $G_{(2)}^{13} = 0$,

$$G_{(2)}^{13} = -\frac{1}{4} \frac{P m^{-1} H_2 \widetilde{R} a}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + 2 \widetilde{R} a}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{R} a|^2} - \frac{2}{\widetilde{R} a} \right\} + \\ + \frac{i}{4} \frac{(1 - W_2) H_2 \widetilde{R} a}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* - \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2}}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{R} a|^2} \right\} \quad (143)$$

$$G_{(1)}^{31} = \frac{i P m^{-1} H_1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \times \\ \times \{ (A_1 A_3^* - A_3 A_1^*) - \hat{a} A_3^* (e_3 A_1 - e_1 A_3) - \hat{a}^* A_3 (e_1 A_3^* - e_3 A_1^*) \} - \\ - \frac{(1 - W_1) H_1}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \{ (A_1 A_3^* + A_3 A_1^*) - \hat{a} A_3^* (e_3 A_1 + e_1 A_3) - \\ - \hat{a}^* A_3 (e_1 A_3^* + e_3 A_1^*) + 2 |\hat{a}|^2 |A_3|^2 e_1 e_3 \}$$

$$-\hat{a}^* A_3 (e_1 A_3^* + e_3 A_1^*) + 2|\hat{a}|^2 |A_3|^2 e_1 e_3 \} \quad (144)$$

because $e_1 = 0$ and $A_1 = A_1^* = 0$;

$$\begin{aligned} G_{(2)}^{31} = & \frac{1}{4} \frac{Pm^{-1} H_2 \widetilde{Ra}}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + 2\widetilde{Ra}}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}|^2} - \frac{2}{\widetilde{Ra}} \right\} + \\ & + \frac{i}{4} \frac{(1 - W_2) H_2 \widetilde{Ra}}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* - \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2}}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}|^2} \right\} \end{aligned} \quad (145)$$

$$\begin{aligned} G_{(1)}^{23} = & \frac{1}{4} \frac{Pm^{-1} H_1 \widetilde{Ra}}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_{123}}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* + \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + 2\widetilde{Ra}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} - \frac{2}{\widetilde{Ra}} \right\} - \\ & - \frac{i}{4} \frac{(1 - W_1) H_1 \widetilde{Ra}}{|\hat{q}_{12}|^2 |\hat{D}_{W_{12}}|^2 |\hat{D}_{H_{12}}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* - \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} \right\} \end{aligned} \quad (146)$$

$$\begin{aligned} G_{(2)}^{23} = & \frac{i Pm^{-1} H_2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \times \\ & \times \left\{ (B_3 B_2^* - B_2 B_3^*) - \hat{b} B_3^* (e_2 B_3 - e_3 B_2) - \hat{b}^* B_3 (e_3 B_2^* - e_2 B_3^*) \right\} - \\ & - \frac{(1 - W_2) H_2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ (B_3 B_2^* + B_2 B_3^*) - \hat{b} B_3^* (e_2 B_3 + e_3 B_2) - \right. \\ & \left. - \hat{b}^* B_3 (e_3 B_2^* + e_2 B_3^*) + 2|\hat{a}|^2 |B_3|^2 e_2 e_3 \right\} = 0 \end{aligned} \quad (147)$$

because $e_2 = 0$ and $B_2 = B_2^* = 0$;

$$\begin{aligned} G_{(1)}^{32} = & -\frac{1}{4} \frac{Pm^{-1} H_1 \widetilde{Ra}}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_{123}}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* + \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + 2\widetilde{Ra}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} - \frac{2}{\widetilde{Ra}} \right\} - \\ & - \frac{i}{4} \frac{(1 - W_1) H_1 \widetilde{Ra}}{|\hat{q}_{12}|^2 |\hat{D}_{W_{12}}|^2 |\hat{D}_{H_{12}}|^2} \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* - \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} \right\} \end{aligned} \quad (148)$$

$$\begin{aligned}
G_{(2)}^{32} &= \frac{iPm^{-1}H_2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \times \\
&\times \left\{ (B_2 B_3^* - B_3 B_2^*) - \hat{b} B_3^* (e_3 B_2 - e_2 B_3) - \hat{b}^* B_3 (e_2 B_3^* - e_3 B_2^*) \right\} - \\
&- \frac{(1 - W_2)H_2}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \left\{ (B_2 B_3^* + B_3 B_2^*) - \hat{b} B_3^* (e_2 B_3 + e_3 B_2) - \right. \\
&\quad \left. - \hat{b}^* B_3 (e_3 B_2^* + e_2 B_3^*) + 2|\hat{a}|^2 |B_3|^2 e_2 e_3 \right\} \quad (149)
\end{aligned}$$

For the correlators components obtained here we use the following relationships:

$$\begin{aligned}
\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} - \hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* &= 2i (1 + Pr^{-1}) (1 - W_2) + \\
&+ \tilde{Q} H_2^2 \left(\frac{2i (Pm^{-1} - Pr^{-1}) (1 - W_2)}{Pm^{-2} + (1 - W_2)^2} \right) \quad (150)
\end{aligned}$$

$$\begin{aligned}
\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} &= 2 (Pr^{-1} - (1 - W_2)^2) + \\
&+ \frac{2\tilde{Q} H_2^2 (Pr^{-1} Pm^{-1} + (1 - W_2)^2)}{Pm^{-2} + (1 - W_2)^2} \quad (151)
\end{aligned}$$

$$\begin{aligned}
&\left| \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \tilde{R}a \right|^2 = \\
&= |\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{\theta_2}|^2 + \tilde{R}a \left(\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} \right) + \tilde{R}a^2 = \\
&= \left[\frac{\left(Pm^{-1} - (1 - W_2)^2 + \tilde{Q} H_2^2 \right)^2 + (1 - W_2)^2 (1 + Pm^{-1})^2}{(1 + (1 - W_2)^2) (Pm^{-2} + (1 - W_2)^2)} \right] \times \\
&\quad \times (1 + (1 - W_2)^2) (Pr^{-2} + (1 - W_2)^2) + \tilde{R}a^2 + \\
&+ 2\tilde{R}a \left[(Pr^{-1} - (1 - W_2)^2) + \frac{\tilde{Q} H_2^2 (Pr^{-1} Pm^{-1} + (1 - W_2)^2)}{Pm^{-2} + (1 - W_2)^2} \right]; \quad (152)
\end{aligned}$$

Let us substitute equations (150)-(152) in expressions for the components of $T_{(2)}^{31}$ and $T_{(1)}^{32}$. As a result we obtain:

$$T_{(2)}^{31} = - \frac{\tilde{R}a (1 + Pm^2 \tilde{W}_2^2) \tilde{W}_2 \left[\left((1 + Pr) (1 + Pm^2 \tilde{W}_2^2) \right) + Q H_2^2 (Pr - Pm) \right]}{2 \left[(1 - Pm \tilde{W}_2^2 + Q H_2^2)^2 + \tilde{W}_2^2 (1 + Pm)^2 \right]} \times$$

$$\begin{aligned}
& \times \left[\left(\left(1 - Pm\widetilde{W}_2^2 + QH_2^2 \right)^2 + \widetilde{W}_2^2(1 + Pm)^2 \right) \left(1 + Pr^2\widetilde{W}_2^2 \right) + \right. \\
& + 2Ra \left(\left(1 - Pr\widetilde{W}_2^2 \right) \left(1 + Pm^2\widetilde{W}_2^2 \right) + QH_2^2 \left(1 + Pm\widetilde{W}_2^2 \right) \right) + \\
& \left. + Ra^2(1 + Pm^2\widetilde{W}_2^2) \right]^{-1} = -\alpha^{(2)} \cdot \widetilde{W}_2 \quad (153)
\end{aligned}$$

$$\begin{aligned}
T_{(1)}^{32} = & \frac{\widetilde{Ra}(1 + Pm^2\widetilde{W}_1^2)\widetilde{W}_1 \left[\left((1 + Pr)(1 + Pm^2\widetilde{W}_1^2) \right) + QH_1^2(Pr - Pm) \right]}{2 \left[(1 - Pm\widetilde{W}_1^2 + QH_1^2)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right]} \times \\
& \times \left[\left(\left(1 - Pm\widetilde{W}_1^2 + QH_1^2 \right)^2 + \widetilde{W}_1^2(1 + Pm)^2 \right) \left(1 + Pr^2\widetilde{W}_1^2 \right) + \right. \\
& + 2Ra \left(\left(1 - Pr\widetilde{W}_1^2 \right) \left(1 + Pm^2\widetilde{W}_1^2 \right) + QH_1^2 \left(1 + Pm\widetilde{W}_1^2 \right) \right) + \\
& \left. + Ra^2(1 + Pm^2\widetilde{W}_1^2) \right]^{-1} = \alpha^{(1)} \cdot \widetilde{W}_1 \quad (154)
\end{aligned}$$

here $\widetilde{W}_1 = 1 - W_1$, $\widetilde{W}_2 = 1 - W_2$; $\alpha^{(1)}$ and $\alpha^{(2)}$ are coefficients of nonlinear hydrodynamic α -effect in an electrically conductive medium with temperature stratification. Comparing the expressions (133) and (139), and (135) and (140), we find the connection of $S_{(2)}^{31}$ with $T_{(2)}^{31}$ and $S_{(1)}^{32}$ with $T_{(1)}^{32}$, i.e.

$$S_{(2)}^{31} = \frac{H_2^2 T_{(2)}^{31}}{Pm^{-2} + \widetilde{W}_2^2}; \quad S_{(1)}^{32} = \frac{H_1^2 T_{(1)}^{32}}{Pm^{-2} + \widetilde{W}_1^2} \quad (155)$$

To close the equations of large-scale magnetic field (89), we need to calculate the turbulent e.m.f. $G_{(2)}^{13} - G_{(2)}^{31}$ and $G_{(1)}^{23} - G_{(1)}^{32}$. Taking into account the equations (143), (145), (146), (148) we obtain:

$$\begin{aligned}
\delta G_{(2)} = G_{(2)}^{13} - G_{(2)}^{31} = & -\frac{1}{2} \frac{Pm^{-1} H_2 \widetilde{Ra}}{|\hat{q}_2|^2 |\hat{D}_{W_2}|^2 |\hat{D}_{H_2}|^2} \times \\
& \times \left\{ \frac{\hat{q}_2^* \hat{D}_{W_2}^* \hat{D}_{\theta_2}^* + \hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + 2\widetilde{Ra}}{|\hat{q}_2 \hat{D}_{W_2} \hat{D}_{\theta_2} + \widetilde{Ra}|^2} - \frac{2}{\widetilde{Ra}} \right\} \quad (156)
\end{aligned}$$

$$\delta G_{(1)} = G_{(1)}^{23} - G_{(1)}^{32} = \frac{1}{2} \frac{Pm^{-1} H_1 \widetilde{Ra}}{|\hat{q}_1|^2 |\hat{D}_{W_1}|^2 |\hat{D}_{H_1}|^2} \times$$

$$\times \left\{ \frac{\hat{q}_1^* \hat{D}_{W_1}^* \hat{D}_{\theta_1}^* + \hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + 2\widetilde{Ra}}{|\hat{q}_1 \hat{D}_{W_1} \hat{D}_{\theta_1} + \widetilde{Ra}|^2} - \frac{2}{\widetilde{Ra}} \right\} \quad (157)$$

After substituting the expressions (151)-(152) in the equations (156)-(157), we obtain the expression:

$$\delta G_{(2)} = \frac{Pm H_2}{((1 - Pm \widetilde{W}_2^2 + QH_2^2)^2 + \widetilde{W}_2^2 (1 + Pm)^2)} \times$$

$$\times \left\{ 1 - Ra \left[(1 - Pr \widetilde{W}_2^2) + \frac{QH_2^2 (1 + Pr Pm \widetilde{W}_2^2)}{(1 + Pm^2 \widetilde{W}_2^2)} + Ra \right] \times \right.$$

$$\times \left[\left((1 - Pm \widetilde{W}_2^2 + QH_2^2)^2 + \widetilde{W}_2^2 (1 + Pm)^2 \right) \frac{(1 + Pr^2 \widetilde{W}_2^2)}{(1 + Pm^2 \widetilde{W}_2^2)} + \right.$$

$$\left. \left. + 2Ra \left[(1 - Pr \widetilde{W}_2^2) + \frac{QH_2^2 (1 + Pr Pm \widetilde{W}_2^2)}{(1 + Pm^2 \widetilde{W}_2^2)} \right] + Ra^2 \right]^{-1} \right\} =$$

$$= \alpha_H^{(2)} \cdot H_2; \quad (158)$$

$$G_{(1)} = \frac{-Pm H_1}{((1 - Pm \widetilde{W}_1^2 + QH_1^2)^2 + \widetilde{W}_1^2 (1 + Pm)^2)} \left\{ 1 - \right.$$

$$\left. - Ra \left[(1 - Pr \widetilde{W}_1^2) + \frac{QH_1^2 (1 + Pr Pm \widetilde{W}_1^2)}{(1 + Pm^2 \widetilde{W}_1^2)} + Ra \right] \times \right.$$

$$\times \left[\left((1 - Pm \widetilde{W}_1^2 + QH_1^2)^2 + \widetilde{W}_1^2 (1 + Pm)^2 \right) \frac{(1 + Pr^2 \widetilde{W}_1^2)}{(1 + Pm^2 \widetilde{W}_1^2)} + \right.$$

$$\left. \left. + 2Ra \left[(1 - Pr \widetilde{W}_1^2) + \frac{QH_1^2 (1 + Pr Pm \widetilde{W}_1^2)}{(1 + Pm^2 \widetilde{W}_1^2)} \right] + Ra^2 \right]^{-1} \right\} =$$

$$= -\alpha_H^{(1)} \cdot H_1; \quad (159)$$

Here $\alpha_H^{(1)}$, $\alpha_H^{(2)}$ are coefficients of nonlinear MHD α -effect in an electrically conductive medium with temperature stratification. The coefficients of the nonlinear MHD α -effect is responsible for the generation of large-scale magnetic fields and consist of two parts:

$$\alpha_H^{(1)} = \alpha_H^{(0)}(1 - Ra \cdot \Phi(W_1, H_1)), \quad \alpha_H^{(2)} = \alpha_H^{(0)}(1 - Ra \cdot \Phi(W_2, H_2)) \quad (160)$$

The first part of the $\alpha_H^{(0)}$ is determined only by the action of external helical force \vec{f}_0 , the second part of the coefficients $\alpha_H^{(1)}$ and $\alpha_H^{(2)}$ are associated with the presence of the temperature stratification $Ra \neq 0$ if $\frac{dT_{00}}{dz} \neq 0$. Here $\Phi(W_{1,2}, H_{1,2})$ is a certain function from $W_{1,2}$ and $H_{1,2}$.

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